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SOLUTION OF ORDINARY INTEGRODIFFERENTIAL EQUATIONS BY THE METHOD OF OSCILLATING FUNCTIONS

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Introduction

In this paper, following [1]–[3], we theoretically substantiate in the sense of ([4], Chap. 14; [5], Chap. 1) the method of oscillating functions (subdomains) for ordinary integrodifferential equations and certain their generalizations. In particular, we prove the convergence of the method under the minimal (by now) restrictions on the initial data and formulate effective (including unimprovable by the order) error estimates depending on the structural features of the initial data. Here we essentially use results and denotations of [2], [3].

1. Problem definition

Consider the linear operator equation

$$Ax \equiv x^{(m)}(t) + B(x;t) = y(t), \quad -1 \leqslant t \leqslant 1, \tag{1.1}$$

under the boundary value conditions

$$R_l(x) = 0, \quad l = \overline{0, m-1}. \tag{1.2}$$

Here B is a linear (including integrodifferential) operator, whose range belongs to the space $L_1(-1,1)$, $y \in L_1(-1,1)$, R_l are linearly independent functionals in the space $C^{m-1}[-1,1]$, m is an integer nonnegative value. In addition, conditions (1.2) for $m = 0$ are absent.

We search an approximate solution of problem (1.1)–(1.2) in the form of a polynomial

$$x_n(t) = \sum_{k=1}^{n+m} \alpha_k t^{k-1}, \quad t \in [-1,1], \quad n \in \mathbb{N}, \tag{1.3}$$

whose coefficients $\alpha_k = \alpha_{k,n}$ are defined by the method of subdomains from the system of linear algebraic equations (SLAE)

$$\sum_{k=1}^{n+m} \alpha_k \int_{t_{i-1}}^{t_i} A(t^{k-1};t)dt = \int_{t_{i-1}}^{t_i} y(t)dt, \quad i = \overline{1, n}, \tag{1.4}$$

$$\sum_{k=1}^{n} \alpha_k R_l(t^{k-1}) = 0, \quad l = \overline{0, m-1}. \tag{1.5}$$

Here $\{t_k = t_{k,n}\}_0^n$ is a certain system of nodes from $[-1,1]$. Below we show that the choice of the nodes is essential for the substantiation of the convergence and the estimation of the method error.