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One Class of Simultaneous Pursuit Games

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Abstract—Let A and B be given convex closed bounded nonempty subsets in a Hilbert space H; let the first player choose points in the set A and let the second one do those in the set B. We understand the payoff function as the mean value of the distance between these points. The goal of the first player is to minimize the mean value, while that of the second player is to maximize it. We study the structure of optimal mixed strategies and calculate the game value.

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1. INTRODUCTION

One of the main problems of the general game theory consists in studying the structure of optimal mixed strategies and in calculating the game value. This problem was studied in many papers (e.g., [1–6]) in the case, when the phase space of players is a finite-dimensional Euclidian space. However, simultaneous pursuit games in Hilbert spaces are less studied. In the paper [6] we consider issues of the existence and the structure of ε-optimal and optimal mixed strategies and calculate the game value in the case when both players choose points in one and the same subset of a Hilbert space.

In this paper we develop the results obtained in [6]. We assume that players choose points in different subsets, and the payoff function is the distance between these points. Certainly, in a general case, when proving the existence of optimal strategies by a known scheme one should proceed to mixed strategies (see the exact statement below).

Let A and B be given closed convex bounded nonempty subsets of a separable Hilbert space H. We define a probability measure in H as follows. Since H is isomorphic to the space

$$l_2 = \{x = (x_1, x_2, \dots) : x_1^2 + x_2^2 + \dots < \infty\}$$

with the scalar product $(x, y) = \sum_{i=1}^{\infty} x_i y_i$, it suffices to define a probability measure on l_2.

We denote the Borel algebra of subsets of R^n by the symbol $B(R^n)$. The set

$$J_n(C) = \{x \in l_2 : (x_1, \dots, x_n) \in C \in B(R^n)\}$$

is called a cylindrical subset of the space l_2. One can also treat a cylindrical set $J_n(C)$, $C \in B(R^n)$, as a cylindrical set whose base belongs to $R^n, R^{n+1}, \dots$, because $J_n(C) = J_{n+1}(C \times R)$. Let the symbol $B(l_2)$ stand for the least σ-algebra containing the system of all cylindrical sets.

Let $p_1, p_2, \dots$ be a sequence of probability measures on $(R, B(R)), (R^2, B(R^2)), \dots$ which have the consistency property: $p_{n+1}(C \times R) = p_n(C)$ for any $n = 1, 2, \dots$ and $C \in B(R^n)$. Then in accordance

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