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# Projection Solution Methods for One Nonlinear Singular Integral Equation

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**Abstract**—In this paper we consider a general projection method for the solution of a nonlinear singular integral equation and its applications in the method of orthogonal polynomials, the subdomains method, and the collocation method.

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*Brief communication*

**1. Introduction.** Let $X, Y$ be weighted Lebesgue spaces, $X = \overset{0}{L}_2(p) = \left\{x \in L_2(p) : \int\limits_{-1}^{1} p(t)x(t)dt = 0\right\}$, $p(t) = \frac{1}{\sqrt{1-t^2}}$, $Y = L_2(q)$, $q(t) = \sqrt{1-t^2}$, with norms $\|x\|_X = \sqrt{\int\limits_{-1}^{1} p(t)|x(t)|^2\, dt}$ and $\|y\|_Y = \sqrt{\int\limits_{-1}^{1} q(t)|y(t)|^2\, dt}$, correspondingly. Analogously to [1] (P. 77), the main object of our research is the operator equation

$$K(x) \equiv Sx + \lambda Th(x) = y \quad (x \in X,\ y \in Y), \tag{1.1}$$

where operators $S : X \to Y$ and $T : X \to Y$ are defined by the equalities

$$Sx = \frac{1}{\pi}\int_{-1}^{1} \frac{x(\tau)}{(\tau - t)\sqrt{1-\tau^2}}\, d\tau, \quad Th(x) = \frac{1}{\pi}\int_{-1}^{1} \frac{h(t,\tau,x(\tau))}{\sqrt{1-\tau^2}}\, d\tau;$$

the singular integral $Sx$ is understood as the Cauchy—Lebesgue principal value. Here $y(t)$ and $h(t,\tau,u)$ are known functions in their domains, $\lambda$ is a numerical parameter, and $x(t)$ is the desired function.

We assume that the function $h(t,\tau,u)$ for any $(t,\tau) \in [-1,1]^2$ and $-\infty < u_1, u_2 < +\infty$ satisfies the conditions

$$|h(t,\tau,u_1) - h(t,\tau,u_2)| \le M|u_1 - u_2|, \quad h(t,\tau,0) = 0. \tag{1.2}$$

In what follows we need the theorem (proved in [1], P. 79) on the existence and the uniqueness of a solution to Eq. (1.1).

**Lemma.** *Let a function* $h(t,\tau,u)$ *satisfy conditions* (1.2). *Then for* $|\lambda| < \frac{\sqrt{2}}{M}$ *problem* (1.1) *has a unique solution* $x^* \in X$ *with any right-hand side* $y \in Y$ *and*

$$\|x^*\|_X \le (1-q)^{-1}\|y\|_Y, \quad q = |\lambda|\frac{M}{\sqrt{2}}.$$

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