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CONVERGENCE OF SERIES IN LOCALLY CONVEX SPACES

Yu.F. Korobeĭnik

1. Let H be a locally convex space over a field of scalars Φ (everywhere in what follows either $\Phi = \mathbb{C}$, or $\Phi = \mathbb{R}$) with a set of prenorms $Q = \{q\}$, which determines the topology μ. Further, let B be a certain set of indices and $x(t) : B \to H$ a mapping from B into H. We set $X_B = \{x(t) : t \in B\}$. Let $\{t_k\}_{k=1}^{\infty}$ be a certain sequence of indices from B and $\Lambda := \{t_k\}_{k=1}^{\infty}$, $X_\Lambda = \{x(t_k) : k \geqslant 1\}$. Everywhere in what follows we assume that $q(x(t)) > 0 \ \ \forall t \in B, \forall q \in Q$. Put $\tau_q(t) := \ln q(x(t))$ $\forall t \in B, \forall q \in Q$.

The main objective of this article is the determination of conditions (in the form of criteria when possible) for the convergence of the series

$$\sum_{k=1}^{\infty} c_k x(t_k), \quad c_k \in \Phi, \quad k = 1, 2, \dots \tag{1}$$

Since for every convergent in H series (1) the necessary convergence condition $\lim\limits_{k\to\infty} |c_k| q(x(t_k)) = 0$ $\forall q \in Q$ is fulfilled, the following proposition takes place.

Proposition 1. *If series* (1) *converges in* H, *then*

$$\lim_{k\to\infty} (\ln |c_k| + \tau_q(t_k)) = -\infty \quad \forall q \in Q. \tag{2}$$

Corollary. If series (1) converges in H, then

$$\limsup_{k\to\infty} (\ln |c_k| + \tau_q(t_k)) < +\infty \ \ \forall q \in Q. \tag{3}$$

We will say that a sequence Λ is *nuclear* with respect to the topology μ if

$$\forall q \in Q \ \ \exists q_1 \in Q : \sum_{k=1}^{\infty} \exp\left(\tau_q(t_k) - \tau_{q_1}(t_k)\right) < \infty. \tag{4}$$

Proposition 2. *If* Λ *is nuclear with respect to* μ *and condition* (3) *is fulfilled, then series* (1) *converges absolutely in* H.

To prove Proposition it suffices to fix arbitrarily $q \in Q$ and choose $q_1 \in Q$ so that condition (4) be valid. Then by virtue of (3) and (4) we have

$$\sum_{k=1}^{\infty} |c_k| q(x(t_k)) = \sum_{k=1}^{\infty} \exp\left(\ln |c_k| + \tau_{q_1}(t_k)\right) \exp\left(\tau_q(t_k) - \tau_{q_1}(t_k)\right) < \infty.$$

Corollary of Proposition 1 and Proposition 2 give us the following

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