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The Ricci Flow on the Two Ball with a Rotationally Symmetric Metric

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Abstract—In this paper we study a boundary-value problem for the Ricci flow in the two-dimensional ball endowed with a rotationally symmetric metric of positive Gaussian curvature and prove short and long time existence results. We construct families of metrics for which the flow uniformizes the curvature along a sequence of times. Finally, we show that if the initial metric has positive Gaussian curvature and the boundary has positive geodesic curvature then the flow uniformizes the curvature along a sequence of times.

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1. INTRODUCTION

Let M be a compact two-dimensional manifold with boundary, and $g(t)$ be a one-parameter family of metrics on M (the parameter t will be called "time"). Let us denote by $R(t)$ the scalar curvature of $g(t)$, and by $k_g(t)$ the geodesic curvature of the boundary ∂M with respect to the outward unit normal. Then one can consider the following boundary-value problem for the Ricci flow on M:

$$\begin{cases} \frac{\partial g}{\partial t} = -Rg & \text{in } M \times (0,T), \\ k_g = \psi & \text{on } \partial M \times (0,T), \\ g(\cdot,0) = g_0. \end{cases} \tag{1}$$

In [1] S. Brendle proved the following result.

Theorem 1. *Let M be a compact surface with boundary ∂M. Then for every initial metric with vanishing geodesic curvature at the boundary, the initial boundary-value problem* (1) *has a unique solution. The solution to the normalized flow is defined for all $t \geq 0$. For $t \to \infty$ the solution converges exponentially to a metric with constant Gauss curvature and vanishing geodesic curvature.*

In the present paper we consider the following boundary-value problem for a one-parameter family of metrics $g(t)$ on the two-dimensional ball $\mathbf{B}^2$:

$$\begin{cases} \frac{\partial g}{\partial t} = -Rg & \text{in } B^2 \times (0,T), \\ k_g = k_0 & \text{on } \partial B^2 \times (0,T), \\ g(\cdot,0) = g_0, \end{cases} \tag{2}$$

where g_0 is a rotationally symmetric metric on $\mathbf{B}^2$:

$$ds^2 = dr^2 + f(r)^2 d\omega^2,$$

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