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Evolutional Equations with Singularities in Generalized Stepanov Spaces

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Let F and U be metric spaces with the corresponding metrics ρ_F and ρ_U. According to Hadamard ([1], P. 10), the problem of finding a solution $u \in U$ to the equation $Au = f$ with given $f \in F$ is said to be well-posed on spaces (F, U), if the following conditions are fulfilled:

a) for any $f \in F$ a solution $u \in U$ to the mentioned equation exists,

b) the solution is defined uniquely,

c) the problem is stable on spaces (F, U), i. e., for any $\varepsilon > 0$ one can choose $\delta > 0$ such that the inequality $\rho_F(f_1, f_2) < \delta$ yields $\rho_U(u_1, u_2) < \varepsilon$.

The stability of the problem depends on the chosen topologies in U and F; choosing the topologies in a proper way, one can formally obtain the continuity of the operator A^{-1}, whose existence is guaranteed by conditions a) and b).

In this connection we have the following problem on the choice of topologies in the space F (of problem data) and the space U (of problem solutions):

1. these topologies have to be independent of the operator A,

2. it is desired to have the widest class of spaces of the initial data $f \in F$, with which the problem solution $u \in U$ keeps "good properties."

In applied problems, the topologies of the following normalized spaces of functions $f(x)$, $x \in \Omega \subset R^1$, are used most often: $C(\Omega)$ is the space of continuous and bounded in Ω functions with the norm

$$\|f\|_C = \sup_{x\in\Omega} |f(x)|;$$

$C^{(l)}(\Omega)$ is the space of functions which are continuous and bounded together with their derivatives up to the order l ($l \in N$),

$$C^{(l)}(\Omega) = \left\{ f(x) : f^{(k)}(x) \in C(\Omega),\ \|f\|_{C^{(l)}} = \sum_{k=0}^{l} \|f^{(k)}\|_C,\ l = 1, 2, \dots \right\},$$

$L_p(\Omega)$ is the space of integrable with the degree $p \geq 1$ functions with the norm

$$\|f\|_{L_p} = \left[\int_\Omega |f(x)|^p dx \right]^{\frac{1}{p}}.$$

Depending on the problem, the corresponding weight spaces are also used.

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