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A Multisheet Plane Figure and its Medial Axis

I. S. Mekhedov[1*]

[1]Dorodnitsyn Computing Center of the Russian Academy of Sciences, ul. Vavilova 40, Moscow, 119333 Russia

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Abstract—A multisheet plane figure is a projection onto the Euclidean plane of a surface whose every point has a neighborhood biuniquely projectable onto the plane. The notion of a multisheet plane figure is a generalization of the usual notion of a plane figure for the case when the boundary contains self-intersecting curves and pairwise intersecting ones. The medial axis of a usual plane figure is the set of points equidistant from the boundary. In this paper we introduce the notion of the medial axis of a multisheet plane figure and study its properties. Medial axes have numerous applications in engineering and computer science.

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1. INTRODUCTION

The goal of this paper is the definition and the study of notions of a *multisheet plane figure* and the *medial axis of a multisheet plane figure*.

A multisheet plane figure is a generalization of a plane figure to the case when bounding curves are self-intersecting or intersect each other. If the boundary of a multisheet plane figure represents a projection of the boundary of some two-dimensional manifold with an edge onto the plane, then the figure admits a natural interpretation. Moreover, we impose certain constraints on the "generating" manifold itself. We require that each its point should have a neighborhood such that any vertical straight line intersects it no more than at one point.

Our interest to multisheet plane figures is connected with the definition of the medial axis for this class of objects.

Let Ω be a connected open bounded subset in $\mathbb{R}^n$. For each point $x \in \Omega$ we denote by $\mathcal{B}(x)$ the set of nearest to x boundary points of the set Ω^c. i.e., $\mathcal{B}(x) = \{y \in \Omega^c \mid d(x,y) = d(x,\Omega^c)\}$, where $\Omega^c = \mathbb{R}^n \setminus \Omega$ is the supplement of the set Ω, and d is the usual Euclidian distance. Note that $\mathcal{B}(x)$ is nonempty for any point $x \in \Omega$, because the set Ω^c is closed and Ω is bounded.

Definition 1. The *medial axis*[1)] [3] of a connected open bounded set $\Omega \subseteq \mathbb{R}^n$ is the set $\mathcal{M}_\Omega$ of points $x \in \Omega$ which have at least two nearest boundary points, i.e., $\mathcal{M}_\Omega = \{x \in \Omega \mid \mathrm{Card}(\mathcal{B}(x)) \geq 2\}$.

In the case $n = 2$, for practical reasons, when considering plane bounded domains, one usually imposes certain constraints on the boundary of the set $\Omega^c \in \mathbb{R}^2$; in this case the set $\overline{\Omega}$ is called a plane figure.

Definition 2. A *plane figure* is a connected closed plane domain bounded by a finite number of embedded pairwise disjoint Jordan curves [2].

*E-mail: mehedov@mail.ru.

1)The term "the medial axis" was introduced by H. Blum [1] in 1967 for plane figures. In the Russian publications one usually uses the notion of the *skeleton* of a plane figure [2]. The skeleton of a plane figure is the set of centers of *maximal empty circles* that belong to this figure; the skeleton represents the closure of the medial axis of the figure.