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CS-Rickart modules

A. N. Abyzov* and T. H. N. Nhan**

Kazan (Volga Region) Federal Univesrity, ul. Kremlyovskaya 18, Kazan, 420008 Russia

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Abstract—We introduce a notion of CS-Rickart module being a modular analog of the ACS-ring concept. We describe the rings over which each finitely generated projective module is CS-Rickart module. The presented results yield the known results related to Rickart modules and semihereditary rings.

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Throughout this paper, all rings are assumed to be associated with a nonzero unity element, and all modules are assumed to be unitary right modules.

Let M be a right R-module. M is called a *Rickart module* if for every $\varphi \in S = \mathrm{End}_R(M)$ $\mathrm{Ker}\,\varphi = eM$ for some $e^2 = e \in S$. A ring R is called a right *Rickart ring* (or a *p. p.-ring*) if every principal right ideal of R is projective. A module M is called a *d-Rickart module* (or a *dual Rickart module*) if for every $\varphi \in S = \mathrm{End}_R(M)$ $\mathrm{Im}\varphi = eM$ for some $e^2 = e \in S$.

A module is called a *CS-module* if every its submodule is essential in a direct summand. We say that a submodule N of a module M *lies above a direct summand* of M if there is a decomposition $M = N_1 \oplus N_2$ such that $N_1 \subset N$ and $N_2 \cap N$ small in N_2. A module M is called a *d-CS-module* (or a *lifting module*) if every submodule of M lies above a direct summand of M.

A module M is called a *CS-Rickart module* if $\mathrm{Ker}\,\varphi$ is essential in a direct summand of M for every $\varphi \in S = \mathrm{End}_R(M)$. A ring R is called a right *CS-Rickart ring* (or *ACS-ring*) if the right annihilator of every element of R is an essential submodule of a direct summand of R_R. A module M is called a *d-CS-Rickart module* if $\mathrm{Im}\,\varphi$ lies above a direct summand of M for every $\varphi \in S = \mathrm{End}_R(M)$.

In what follows, the notations $N \trianglelefteq M$, or $N \ll M$ mean that N is an *essential* (or *large*) submodule of M, or N is a *superfluous* (or *small*) submodule of M, respectively. The largest singular submodule of M will be denoted by $Z(M)$.

Rickart modules have been studied in [1–3] while CS-Rickart rings have been studied in [4–6]. Besides, the paper [2] has described the rings over which each finitely generated projective right module as a Rickart module.

Lemma 1. (1) *Every direct summand of a CS-Rickart module is a CS-Rickart module.*

(2) *Every direct summand of a d-CS-Rickart module is a d-CS-Rickart module.*

Lemma 2. *Let A be a uniform hereditary right R-module, B be a singular uniform Artinian right R-module. Then the following statements hold:*

(1) *$A \oplus B$ is a CS-Rickart module.*

(2) *If B is not an A-injective module, then $A \oplus B$ is not a CS-module.*

*E-mail: Adel.Abyzov@kpfu.ru.
**E-mail: tranhoaingocnhan@gmail.com.