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On Arithmetical Level of the Class of Superhigh Sets

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Abstract—We determine the proper arithmetical level of the class of superhigh sets.

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A set $A \subseteq \mathbb{N}$ is called *superhigh* if $\varnothing'' \leqslant_{tt} A'$. Superhigh sets were introduced and first investigated by Mohrherr in [1] (see also [2, 3]). Miller and Hirschfield proved ([3], P. 190) that for every Σ^0_3 null class there exists a non-computable c. e. (computably enumerable) set which is computable in every Martin-Löf random sequence from this class. This result attracted an interest to questions about arithmetical levels of classes of high sets. Consider the operator which maps every class $\mathcal{H} \subseteq 2^{\mathbb{N}}$ to the collection of c. e. sets

$$\mathcal{H}^{\diamond} = \{A : A \text{ is c. e. and } \forall X \in \mathcal{H}\,[X \text{ is Martin-Löf random} \Rightarrow A \leqslant_T X]\}.$$

This operator is called the diamond operator. The values of this operator on classes of superhigh sets (Shigh) and almost everywhere dominating sets (AED, see [4]) were studied in [3] (§ 8.5). Note that AED is the Σ^0_3-class and Shigh is contained in a null Σ^0_3-class (see ([3], § 8.5). In the same work it was shown that $\mathrm{SJT}_{\text{c. e.}} \subseteq \mathrm{Shigh}^{\diamond} \subset \mathrm{AED}^{\diamond}$, where $\mathrm{SJT}_{\text{c. e.}}$ is the class of strongly jump traceable c. e. sets [3, 5] (for details on traceability see [6]). Later N. Greenberg, A. Nies, and D. Hirschfield [7] established that $\mathrm{Shigh}^{\diamond} = \mathrm{SJT}_{\text{c. e.}}$. However, the question whether the Shigh belongs to the Σ^0_3-level remained open. In this paper we show that $\mathrm{Shigh} \notin \Sigma^0_3$.

We use the notions from the books [3] and [8]. If C is an alphabet, then denote by C^* the set of all finite strings over C. Let $\varnothing$ be the empty string. For strings σ and ρ we write $\sigma \preceq \rho$ if σ is the prefix of ρ; write $\sigma \prec \rho$ if $\sigma \preceq \rho$ and $\sigma \neq \rho$. Denote by σ the length of a string σ. For a set X and numbers $m < n$ denote by $X \upharpoonright [m, n)$ the string σ of length $n - m$ such that the ith ($i < n - m$) symbol of σ equals $X(m + i)$. Instead of $X \upharpoonright [0, n)$ we simply write $X \upharpoonright n$. If σ is a string in $\{0, 1\}^*$, then we write $\sigma \prec X$ if there is an n such that $\sigma \preceq X \upharpoonright n$. Given a string $\sigma \in \{0, 1\}^*$ and a number n, let σ^n be the string σ concatinated with itself n times. If $n = 0$, let $\sigma^n = \varnothing$.

Let $\sigma, \rho, \tau \in \{0, 1\}^*$. We say that σ and ρ are splitting of τ if $\tau \prec \sigma, \rho$ and σ, ρ are $\preceq$-incomparable. We say that a partial function

$$T : \{0, 1\}^* \to \{0, 1\}^*$$

is an f-tree if the convergence of one of the values $T(\sigma 0)$ and $T(\sigma 1)$ implies that the values $T(\sigma)$, $T(\sigma 0)$, and $T(\sigma 1)$ converge and the pair $T(\sigma 0)$, $T(\sigma 1)$ is the splitting of $T(\sigma)$. We write $\sigma \in T$ if σ belongs to the domain of T. We say that a set X belongs to an f-tree T or X is an path of T if $X \upharpoonright n$ belongs to T for infinitely many n. Denote by $\mathrm{Paths}\,(T)$ the collection of all paths of T. Given a partial function ψ, let $\operatorname{dom}\psi$ be the domain of ψ. We write $\psi(x)\downarrow$ if $x \in \operatorname{dom}\psi$ and $\psi(x)\uparrow$ otherwise. Denote by $\langle x, y\rangle$ the standard computable bijection from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$.

To prove the main result we provide a characterization of sets which are truth-table reducible to a jump of fixed set.

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