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ONE MIXED PROBLEM FOR DIFFERENTIAL EQUATION OF THE SECOND ORDER WITH RESPECT TO TIME

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1. In a rectangle $[0,1]\times[0,1]$ we consider the equation

$$\frac{\partial^2 u}{\partial t^2}+(-1)^m a(x)\frac{\partial^{2m+1}u}{\partial x^{2m}\partial t}+(-1)^{m+k}b(x)\frac{\partial^{2m+2k}u}{\partial x^{2m+2k}}=f(t,x) \tag{1}$$

with the boundary conditions

$$u(t,0)=u(t,1)=\frac{\partial u(t,0)}{\partial x}=\frac{\partial u(t,1)}{\partial x}=\dots=\frac{\partial^{m-1}u(t,0)}{\partial x^{m-1}}=\frac{\partial^{m-1}u(t,1)}{\partial x^{m-1}}=0, \tag{2}$$

$$\frac{\partial^{m+r}u(t,0)}{\partial x^{m+r}}=\frac{\partial^{m+r}u(t,1)}{\partial x^{m+r}}=\dots=\frac{\partial^{m+r+k-1}u(t,0)}{\partial x^{m+r+k-1}}=\frac{\partial^{m+r+k-1}u(t,1)}{\partial x^{m+r+k-1}}=0 \tag{3}$$

and the initial conditions

$$u(0,x)=u_0(x),\quad \frac{\partial u(0,x)}{\partial t}=u_1(x) \tag{4}$$

for some k, m, and $0\le r\le m+k$. For $k=0$, $m=1$, equation (1) is an equation of sound propagation in a viscous gas.

In the space $L_p(0<x<1)=L_p$ we introduce the operators $A=(-1)^m a(x)\frac{d^{2m}}{dx^{2m}}$ and $B=(-1)^{m+k}b(x)\frac{d^{2m+2k}}{dx^{2m+2k}}$ with the domains of definition

$$\mathcal{D}(A)=\{u(x)\in W_p^{2m},\ u(0)=u(1)=u'(0)=u'(1)=\dots=u^{(m-1)}(0)=u^{(m-1)}(1)=0\},$$
$$\mathcal{D}(B)=\{u(x)\in W_p^{2m+2k},\ u(0)=u(1)=\dots=u^{(m-1)}(0)=u^{(m-1)}(1)=0,$$
$$u^{(m+r)}(0)=u^{(m+r)}(1)=\dots=u^{(m+r+k-1)}(0)=u^{(m+r+k-1)}(1)=0\},$$

respectively. Then problem (1)–(4) reduces to an abstract Cauchy problem

$$u''+Au'+Bu=f(t)\quad(0<t\le 1),\quad u(0)=u_0,\quad u'(0)=u_1 \tag{5}$$

in the Banach space L_p. We will consider the case where $0\le k\le m$. In this situation, equation (1) turns to be correct by Petrovskiĭ and behaves as a parabolic equation in the sense that problem (1)–(4) reduces to the Cauchy problem (5) for the parabolic differential operator equation. A similar problem was studied in [1] (Chap. 5, § 3). However, in [1] boundary conditions were considered with $r=m$, when the operator B represents a product $B=CA$, where C is the $2k$ order differentiation operator with the Dirichlet boundary conditions. In this case, this operator C is the generating operator of the analytic semigroup.

Equation (1) was considered in [2] in the case where $a(x)\equiv 1$, $b(x)\equiv 1$, the first addend has the form $\frac{\partial}{\partial t}\big(h(x)\frac{\partial u}{\partial t}\big)$, where the function $h(x)$ is from $C^{2m}[0,1]$, $h(x)\ge 0$, and satisfies the conditions

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