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RESONANCE BOUNDARY VALUE PROBLEMS FOR ELLIPTIC EQUATIONS WITH DISCONTINUOUS NONLINEARITIES

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Introduction

Let Ω be a bounded domain in R^n with boundary Γ of class $C_{2\alpha}$, $0 < \alpha < 1$, (see [1], p. 23), $Lu(x) \equiv -\sum_{i,j=1}^{n} (a_{ij}(x)u_{x_i})_{x_j} + c(x)u(x)$ a uniformly elliptic differential operator on $\overline{\Omega}$ with coefficients $a_{ij} \in C_{1\alpha}(\overline{\Omega})$, $a_{ij}(x) = a_{ji}(x)$, $c \in C_{0\alpha}(\overline{\Omega})$. We consider the boundary value problem of the form

$$Lu(x) + g(x, u(x)) = p(x), \quad x \in \Omega, \tag{0.1}$$

$$Bu|_\Gamma = 0, \tag{0.2}$$

where the nonlinearity $g(x,u)$ satisfies the condition $(*)$:

the function $g : \Omega \times R \to R$ is Borel (mod 0) (see [2], p. 157), for almost all $x \in \Omega$, the section $g(x,\cdot)$ possesses on R only discontinuities of the first kind, and $g(x,u) \in [g_-(x,u), g_+(x,u)]$, $g_-(x,u) = \liminf_{s\to u} g(x,s)$, $g_+(x,u) = \limsup_{s\to u} g(x,s)$;

$p(x)$ is a function summable (integrable) on Ω; (0.2) is one of the principal boundary conditions

$$u|_\Gamma = 0,$$

$$\left.\frac{\partial u}{\partial n_L}\right|_\Gamma \equiv \sum_{i,j=1}^{n} a_{ij}(x)u_{x_i}\cos(n,x_j)|_\Gamma = 0,$$

$\cos(n,x_j)$ are the directional cosines of the outer normal n to the boundary Γ;

$$\frac{\partial u}{\partial n_L}(x) + \sigma(x)u(x)|_\Gamma = 0, \tag{0.3}$$

the function $\sigma \in C_{1\alpha}(\Gamma)$ (see [1], p. 23) is nonnegative on Γ and is not equal identically to zero.

By the *strong solution* of problem (0.1)–(0.2) a function $u \in W_q^2(\Omega)$, $q \geq 1$, is called which satisfies equation (0.1) for almost all $x \in \Omega$ and for which the trace of $Bu(x)$ on the boundary Γ of the domain Ω is equal to zero.

We investigate the question of the existence of the strong solutions in so-called resonance case where the problem

$$Lu(x) = 0, \quad x \in \Omega, \tag{0.4}$$

$$Bu|_\Gamma = 0 \tag{0.5}$$