Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 9, pp.27–35, 2005 *UDC 517.95*

AVERAGING OF WEAKLY NONLINEAR PARABOLIC OPERATORS IN A PERFORATED CYLINDER

O.A. Matevosyan and I.V. Filimonova

The questions of the averaging theory for perforated areas are investigated in many papers (see [1]–[3]), where the behavior of solutions near the holes is subjected to certain constraints, for example, the boundary conditions. However, such constraints are not necessary for averaging solutions of equations with sufficiently strong nonlinear absorption. A similar situation also occurs in the desingularization theory. In this connection, mention papers [4]–[7]. Questions of averaging for semilinear elliptic problems in perforated areas are studied in [8], [9], using the Kondratiev–Landis estimate ([10], § 1, item 3).

In this paper, we consider a weakly nonlinear divergent parabolic equation of the second order, whose lowest term increases with respect to the unknown function under the power law. We prove that the sequence of solutions in a perforated cylinder converges to a solution in a non-perforated one if radiuses of the removed cylinders tend to zero at rate which depends on the exponent of the lowest term. The proofs of theorems in this paper are based on the obtained in [11] analogous estimate of a solution of a semilinear parabolic equation by the distance from a point to the domain boundary in the corresponding metric.

We use the following denotations:

$$x=(x_1,\dots,x_n)\in\mathbb{R}^n,\quad n>2,\quad u=(u_1,\dots,u_n),\quad u_x=(u_{x_1},\dots,u_{x_n}),$$
$$|u_x|^2=\sum_{i=1}^n (u_{x_i})^2,\quad u_t=\partial u/\partial t,\quad 0\le t<T<\infty;$$

$\Omega\subset\mathbb{R}^n$ is an arbitrary open subset of $\mathbb{R}^n$ with the boundary $\partial\Omega$, $\overline{\Omega}=\Omega\bigcup\partial\Omega$ is the closure of Ω; $Q=\Omega\times(0,T)$ is a cylinder in the space of points $(x,t)\in\mathbb{R}^{n+1}$, $x\in\Omega$, $t\in[0,T]$, $0<T<\infty$;

$S(Q)=\partial\Omega\times[0,T]$ is the lateral surface of the cylinder $Q\subset\mathbb{R}^{n+1}$; $\Gamma(Q)=S(Q)\cup\Upsilon_0(\Omega)$ is the parabolic boundary of the cylinder $Q\subset\mathbb{R}^{n+1}$, where $\Upsilon_t(\Omega)=\overline{\Omega}\times\{t\}\quad\forall t\in[0,T]$;

$Q(x_0,t_0,R,T)=B(x_0,R)\times[t_0,t_0+T]$ is a cylinder in $\mathbb{R}^{n+1}$ of the radius R and the height T, where $B(x_0,R)=\{x\in\mathbb{R}^n : |x-x_0|\le R\}$ is a closed ball in $\mathbb{R}^n$ of the radius R with the center at the point x_0;

$\mathcal{C}(Q)$ is the totality of continuous in Q functions;

$L_{p,r}(Q)$ is the space which consists of all measurable on Q functions with the finite norm

$$\|u;L_{p,r}(Q)\|=\left(\int_0^T\left(\int_\Omega |u(x,t)|^p dx\right)^{r/p}dt\right)^{1/r},\quad p\ge 1,\quad r\ge 1;$$

we denote $L_{p,r}(Q)$ as $L_p(Q)$ and we do the norm $\|\cdot;L_{p,r}(Q)\|$ as $\|\cdot;L_p(Q)\|$ for $p=r$; $W_p^{1,1}(Q)$ is the Sobolev space consisting of the elements of $L_p(Q)$ which have the generalized derivatives in x and in t with the norm $\|u;W_p^{1,1}(Q)\|=\|u;L_p(Q)\|+\|u_x;L_p(Q)\|+\|u_t;L_p(Q)\|$;