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To the Theory of Boundary-Value Problems for Elliptic Equations with Superposition Operators in the Boundary Condition. I

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Abstract—We study the existence, uniqueness, and constant sign property of classical solutions to a nonlocal boundary-value problem for a second-order elliptic equation in a bounded domain of the Euclidean space. Using the system of maps that define superposition operators, we construct some subset of the domain boundary and establish the connection between the solvability of the problem under consideration and the solvability of the boundary value equation on the constructed subset.

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1. DENOTATIONS AND AUXILIARY RESULTS

We study the following nonlocal boundary-value problem:

$$Lu \equiv \sum_{i,j=1}^{n} a_{ij}(x)u_{x_ix_j}(x) + \sum_{i=1}^{n} b_i(x)u_{x_i}(x) + c(x)u(x) = f(x), \ \ x \in D, \tag{1}$$

$$Bu \equiv u(x) - \sum_{j=1}^{k} \beta_j(x)u(\sigma_j x) = \psi(x), \ \ x \in S. \tag{2}$$

Here and below we use the following denotations (all values are assumed to be real): $x = (x_1, \dots, x_n)$ is a point in a Euclidean space R^n, $n \geq 2$; $|x| = \sqrt{x_1^2 + \cdots + x_n^2}$; D is a *bounded* domain in R^n with the boundary S, $\overline{D} = D \cup S$; $\sigma_1, \dots, \sigma_k$ are maps of $\overline{D}$ to $\overline{D}$. If e is a nonempty set in R^n, $\varepsilon = \text{const} > 0$, then we put $e^\varepsilon = \{x \in S : \rho(x, e) < \varepsilon\}$, where $\rho(x, e) = \inf\limits_{y \in e} |x - y|$; if $e = \emptyset$, then $e^\varepsilon = \emptyset$. We denote by $\varkappa_e(x)$ the characteristic function of the set e, i.e., $\varkappa_e(x) = 1$ for $x \in e$, $\varkappa_e(x) = 0$ for $x \in R^n \setminus e$. We assume that the correlation $e_1 \subset e_2$ does not exclude the coincidence of the sets e_1 and e_2.

Let $J = \{1, \dots, k\}$ be the set of first k natural numbers, $k \geq 2$ (for example, the denotations $\sum\limits_{j=1}^{k} c_j$ and $\sum\limits_{j \in J} c_j$ are equivalent). We denote by J_{lr} combinations of l elements of the set J; here $l = 1, \dots, k$ and $r = 1, \dots, C_k^l$. We put $J_{11} = \{1\}, \dots, J_{1k} = \{k\}$, $J_{l1} = \{1, \dots, l\}, \dots, J_{lC_k^l} = \{k - l + 1, \dots, k\}$ for $2 \leq l < k, \dots, J_{kr} = J_{k1} = J$.

In this paper we use the Hölder spaces $C_{m+\alpha} \equiv C_{m+\alpha}(D)$ ([1], pp. 111, 112). Let us recall certain important facts. The inclusion $z(x) \in C_\alpha(D)$ means, in particular, that $z(x)$ is bounded in $\overline{D}$, and $z(x) \in C_{2+\alpha}(D)$ implies the continuity of $z(x)$ in $\overline{D}$. We denote by C, C^m, and L_p spaces of continuous, m times continuously differentiable, and summable with degree p functions, correspondingly. Moreover, we use S. L. Sobolev spaces $W_p^2(D)$, understanding an element of such a space with $p > n/2$ not as a

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