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H-PROJECTIVELY EQUIVALENT RIEMANNIAN CONNECTIONS

D.A. Kalinin

Introduction

Both the study of projective mappings of Riemannian manifolds and the generalizations of these mappings form a classical problem in the differential geometry. In the present article we consider the Kähler manifolds of arbitrary dimension, which admit so-called H-projective mappings (see [1]–[5]).

In [1], the connection forms of the H-projectively equivalent Riemannian connections of the four-dimensional Kähler manifolds were found. In the investigation the canonical types of bilinear forms on a Riemannian manifold found by A.Z. Petrov in [6] were used. The canonical types indicated above allow to apply the method of moving skew-normal frame (skew-frame, see [7], [8]).

The method of moving skew-frame is one of the basic tools for the study of the pseudo-Riemannian manifolds admitting projective mappings (see [8], [9]). However, even in the four-dimensional case, this method leads to significant computational difficulties in the investigation of the H-projective mappings of the Kähler manifolds and, in particular, in the determination of the H-projectively equivalent Riemannian connections of the Kähler manifolds.

This leads to the necessity to apply the canonical types of bilinear forms which reflect the structure of a Kähler manifold M; namely, the fact that T_pM, $p \in M$, is a complex linear space. Canonical types of that kind were found in [10]. In the present article, by using the types mentioned above, we determine the H-projectively equivalent Riemannian connections on a Kähler manifold (M, g) of arbitrary both dimension and signature.

We use the notation adopted in [10].

1. H-projective mappings of Kähler manifolds

Let (M, J) be a $2n$-dimensional integrable almost complex manifold. Recall that a Kähler metric on M is a (pseudo-)Riemannian metric g such that $g(JX, JY) = g(X, Y)$ for any vector fields X, Y on M and $\nabla J = 0$, where ∇ is the Levi-Civita connection of g. An almost complex manifold M with a given Kähler metric is called a *Kähler manifold*. A smooth curve γ on a Kähler manifold M is said to be H-planar if its tangent vector χ satisfies the equation

$$\nabla_\chi \chi = a(t)\chi + b(t)J(\chi),$$

where both $a(t)$ and $b(t)$ are smooth functions of t.

Let M and M' be two Kähler manifolds with the metrics g and g' and the complex structures J and J' respectively. A diffeomorphism $f : M \to M'$ is called *an H-projective mapping* if for any H-planar curve γ in M the curve $f \circ \gamma$ is H-planar in M'. If a manifold M admits an H-projective

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