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INVESTIGATION OF A NUMERICAL METHOD FOR SOLVING THE SPECTRAL PROBLEM OF THE THEORY OF DIELECTRIC WAVEGUIDES

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Investigation of spectral problems of the theory of dielectric waveguides and development of numerical methods attract large attention (see, e. g., [1]–[4]). In [5]–[7] numerical methods were suggested for searching eigenwaves of cylindric dielectric waveguides on the base of the representation of their amplitudes as a superposition of potentials of simple and double layers. During the last years in solving a series of spectral problems of the electrodynamics authors successfully apply representation of fields in the form of potentials of a simple layer (see, e. g., [8]–[10]). This enables to shorten essentially computer time.

The present article is devoted to the investigation of a numerical method for searching constants of propagation of surface eigenwaves of cylindric dielectric waveguides with a smooth contour of the cross section, which is based on representation of the desired functions in the form of potentials of simple layer. Under assumption of closeness of the refractive indices of the waveguide and the environment, the problem is reduced to a nonlinear spectral problem for a system of singular integral equations. On the base of the known regularization procedure (see, e. g., [11], p. 14), we construct the system of the Galyorkin method. Zeros of the determinant of the matrix of this system are taken in the capacity of approximate solution of this problem. For investigation of the convergence of the method we use the results of [12]. A similar approach was applied to substantiation of the method for calculation of microband lines in [8], [9].

1. The problem of determination of the constants of propagation of eigen surface waves of a cylindric dielectric waveguide under assumption of closeness of the refractive indices of the waveguide and environment can be reduced (see [1], item 2) to search of values of the parameter β, under which nontrivial solutions $u(x, y)$ of the boundary problem

$$\Delta u + \chi_1^2 u = 0, \quad (x, y) \in S, \tag{1}$$

$$\Delta u + \chi_2^2 u = 0, \quad (x, y) \notin \overline{S}, \tag{2}$$

$$u^+ = u^-, \quad \frac{\partial u^+}{\partial \nu} = \frac{\partial u^-}{\partial \nu}, \quad (x, y) \in C, \tag{3}$$

$$u \quad \text{exponentially decreases as} \quad r = \sqrt{x^2 + y^2} \to \infty \tag{4}$$

exist. Here S is a bounded domain with the boundary C, $\chi_i^2 = k_0^2 n_i^2 - \beta^2$, $k_0^2 = \omega^2 \varepsilon_0 \mu_0$, ε_0 is the electric constant, μ_0 is the magnetic constant, ω is the frequency of electromagnetic oscillations, n_1, n_2 are values of the refractive indices of waveguide and environment, $\partial u/\partial \nu$ is the correct normal derivative, f^+ (f^-) is the limit value of the function f inside (outside) the contour C.