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Closed Fejer Cycles for Inconsistent Systems of Convex Inequalities

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1. INTRODUCTION

In this paper we consider certain questions, concerning the construction of generalized discrete solutions to inconsistent systems of convex inequalities based on the notion of a *closed cycle*. This idea is close to the procedure of the successive (cyclic) projection used in the analysis of an inconsistent system of convex inclusions. Let us adduce the necessary conceptual apparatus, as well as denotations and terms:

$\mathbb{R}^n$ is the n-dimensional Euclidian space;

$\mathbb{R}^n_+ = \{x \in \mathbb{R}^n : x \geq 0\}$;

$\|x\|$ is the Euclidean norm of the vector $x \in \mathbb{R}^n$;

$\arg C$ is the optimal vector of the optimization problem labelled, for example, by the symbol C (or by its number in the text);

$\mathrm{Pr}_M(x) := \arg\min\limits_{y\in M} \|x - y\|$;

$\mathrm{Fix}\,\varphi(x)$ is the set of fixed points of the operator $\varphi(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$;

$\varphi_m\varphi_{m-1}\ldots\varphi_1(x)$ is the superposition of operators $\{\varphi_i(\cdot)\}_{i=1}^m$;

$\ldots := z$ means the assignment of the symbol z to the left-hand side of an expression with the sign "=";

$z := \ldots$ means the assignment of the symbol z to the right-hand side of an expression with the sign "=".

Let us define an M-Fejer operator.

Definition 1.1. An operator $\varphi(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is said to be *M-Fejer*, if

$$\varphi(y) = y, \quad \|\varphi(x) - y\| < \|x - y\| \;\; \forall\, y \in M, \;\; \forall\, x \notin M.$$

Here $M \subset \mathbb{R}^n$, $M \neq \emptyset$.

With fixed M we denote by $\mathcal{F}_M$ the class of all M-Fejer operators and we do by $\overline{\mathcal{F}}_M$ the class of those of them which are continuous.

The following assertions on Fejer operators are known (e.g., [1], Chap. II, § 1).

Proposition 1.1. *If $\mathcal{F}_M \neq \emptyset$, then the set M is convex and closed.*

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