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TWO-CYCLIC TRIANGULAR SKEW-SYMMETRIC ITERATION METHOD FOR SOLVING STRONGLY NONSYMMETRIC SYSTEMS

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Introduction. In this article we suggest a new iteration method which allows to solve effectively strongly nonsymmetric systems of equations, which are a result, for example, of a central-difference approximation of the convection–diffusion equation with prevailing convection. Sufficient conditions for convergence are given.

As is known (see [1]), the iteration methods applied for solving the linear systems

$$Ay = f, \tag{1}$$

can be joined by the common formula

$$B_j \frac{y^{j+1} - y^j}{\tau_j} = -(Ay^j - f), \tag{2}$$

where $\{B_j\}$ is a sequence of nondegenerate matrices, $\{\tau_j\}$ a sequence of real parameters, y^j an approximate solution on j-th iteration. If we introduce the notation $H_j = \tau_j B_j^{-1}$, then the iteration process can be written in an equivalent form

$$y^{j+1} = y^j - H_j(Ay^j - f).$$

Definition (see [2]). Iterative methods possessing the property $H_j = H_{j+s}$ for any $j \geq 0$ and a certain fixed $s \geq 1$ are said to be cyclic.

Any matrix A is decomposable into the sum

$$A = A_0 + A_1,$$

where $A_0 = \frac{1}{2}(A + A^*) = A_0^*$ is the symmetric and $A_1 = \frac{1}{2}(A - A^*) = -A_1^*$ is skew-symmetric components of the matrix A. Any skew-symmetric matrix A_1 can be represented as follows

$$A_1 = K_H + K_B,$$

where K_H and K_B are the strictly lower and strictly upper triangular parts of the matrix A_1, respectively, and for them the equalities are valid $K_H = -K_B^*$, $K_B = -K_H^*$.

The idea proposed in [3] to include into the operator of the method B the triangular parts of the skew-symmetric component A_1 of the initial matrix A allows us to obtain a new class of implicit triangular skew-symmetric iteration methods, which are simultaneously of rather simple structure and are designed for solving systems with nonselfadjoint strongly nonsymmetric[1] matrices.

In [3], a structure of the operator B was suggested satisfying the equality

$$B_1 = \tau A_1, \tag{3}$$

[1] A matrix A is said to be *strongly nonsymmetric* if an a certain norm $\|A_1\| \gg \|A_0\|$.

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