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Generalization of the Newton Method for One Class of Nonconvex Mathematical Programming Problems

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This paper continues the study commenced in [1] and [2], where we consider iterative procedures, finding stationary points of smooth functions on a class of nonconvex sets. We generalize the Newton method applied for the solution of convex programming problems for the case, when constraints are represented as a set-theoretical difference of a convex set and a union of several convex sets. We formulate and prove a proposition on the convergence of the algorithm.

1. PROBLEM DEFINITION AND THE ALGORITHM

Consider the following problem: find a point which satisfies the necessary condition for a local minimum of a function $\varphi(x)$ on a set X in the n-dimensional Euclidean space E^n, where X is a set-theoretical difference of certain sets F and $\bigcup\limits_{i=1}^{l} \operatorname{int} G_i$; here F and G_i, $i = \overline{1,l}$, are convex and closed, the sets of inner points of X and G_i, $i = \overline{1,l}$, are nonempty; $\varphi(x)$ belongs to the class $C^2(Y)$ and is strongly convex on a certain convex set Y which includes X. Let each set G_i, $i = \overline{1,l}$, at any its boundary point x have a unique supporting hyperplane, whose normal is assumed to be outer. This means that for all $y \in G_i$ the unit normal vector $n^i(x)$ satisfies the condition $\langle n^i(x), y - x\rangle \leq 0$. We also assume that with each i the unit normal vector $n^i(x)$ is a continuous vector function at the boundary ∂G_i of the set G_i. The latter means that for any point $x_* \in \partial G_i$ and an arbitrary sequence $\{x_k\}$ which belongs to ∂G_i and converges to x_*, with each $\varepsilon > 0$ one can find a number $k(\varepsilon) \in N$ such that all $k \geq k(\varepsilon)$ meet the inequality $\|n^i(x_k) - n^i(x_*)\| < \varepsilon$.

Below we use the following denotations: $s^i(x)$ is a projection of a point x onto the set G_i, $n^i(x)$ is the unit normal vector of the hyperplane supporting to G_i at the point $s^i(x)$, $\Gamma^i(x) = \{e \in E^n : \langle n^i(x), e - s^i(x)\rangle \geq 0\}$, $P(x) = F \bigcap \Gamma^1(x) \bigcap \Gamma^2(x) \bigcap \cdots \bigcap \Gamma^l(x)$. Projections $s^i(x)$ are defined uniquely, because G_i, $i = \overline{1,l}$, are convex sets of the Euclidean space E^n. Since each G_i, $i = \overline{1,l}$, at any its boundary point x has only one supporting hyperplane, vectors $n^i(x)$ and, therefore, half-spaces $\Gamma^i(x)$, $i = 1, 2, \ldots, l$, are uniquely defined for any $x \in$ X. If with certain i points x and $s^i(x)$ do not coincide, then vectors $n^i(x)$ and $x - s^i(x)$ have the same direction. Consequently, $\langle n^i(x), x - s^i(x)\rangle > 0$, i. e., $x \in \operatorname{int} \Gamma^i(x)$. But if x and $s^i(x)$ coincide, then x belongs to the boundary of $\Gamma^i(x)$. Since $x \in \mathrm{X} \subset F$ and $x \in \Gamma^i(x)$ with each $i = 1, 2, \ldots, l$, we conclude that always $x \in P(x)$.

We propose to solve the stated problem by the following algorithm, constructing successive approximations.

Step 0. Put $k = 0$.

Step 1. Let $x_k \in$ X be the kth approximation.

Step 2. Define points $s^i(x_k), i = \overline{1,l}$.

Step 3. Construct half-spaces $\Gamma^i(x_k)$, $i = \overline{1,l}$.

Step 4. Construct the set $P(x_k)$.