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ON TRIGONOMETRICAL INTERPOLATION OF FUNCTIONS OF m-HARMONIC BOUNDED VARIATION

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In the present article we consider the convergence of the Lagrange trigonometrical interpolation procedure with equidistant points for functions with bounded m-harmonic variation at their continuity points.

First, let us recall respective definition introduced in [1] (see also [2], [3]). Let a function f be defined on the segment $[a,b]$, m being a positive integer. We denote by $\{I_n\}$, $n=1,2,\dots$, a sequence of disjoint intervals $I_n=(a_n,b_n)\subseteq[a,b]$.

Let us put

$$V_{m,H}(f;a,b)=\sup\sum_{n=1}^{\infty}\frac{|\Delta^m f(I_n)|}{n}, \tag{1}$$

where

$$\Delta^m f(I_n)\equiv\Delta^m f(a_n,b_n)=\sum_{\nu=0}^{m}(-1)^{m-\nu}C_m^{\nu}f(a_n+\nu h_n),$$
$$h_n=(b_n-a_n)/m,$$

and the least upper bound is taken in (1) over all possible sequences of intervals $\{I_n\}=\{(a_n,b_n)\}$ satisfying the above-described conditions.

The value $V_{m,H}(f;a,b)$ is called m-harmonic variation of the function f on the segment $[a,b]$. If $V_{m,H}(f;a,b)<\infty$, then we say that f has an m-harmonic bounded variation on $[a,b]$.

For $m=1$, the definition of a function of m-harmonic bounded variation turns into the definition of a function of harmonic bounded variation, which was introduced by D. Waterman in [4].

In what follows ξ will stand for an arbitrary real number. Let us put

$$x_{k,n}\equiv x_{k,n}(\xi)=\xi+2k\pi/(2n+1),\quad k=0,\pm1,\dots;\quad n=0,1,2,\dots,$$
$$t_{k,n}(x)=\frac{\sin((n+1/2)(x-x_{k,n}))}{(2n+1)\sin((x-x_{k,n})/2)}=\frac{(-1)^k\sin((n+1/2)(x-\xi))}{(2n+1)\sin((x-x_{k,n})/2)},\quad k=0,\pm1,\dots;\quad n=0,1,2,\dots \tag{2}$$

We denote by $L_n(f;x)\equiv L_{n,\xi}(f;x)$, $n=1,2,\dots$, a trigonometric polynomial of an order not exceeding n, which coincides with the given 2π-periodic function f at the points $x_{k,n}$, $k=0,\pm1,\dots$ (i.e., the Lagrange trigonometric interpolational polynomial). As it is known (see, e.g., [5], Chap. X, § 1, p. 10),

$$L_n(f;x)=\sum_{k=-n}^{n}f(x_{k,n})t_{k,n}(x).$$

In order to prove the main result we need