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GROUP ANALYSIS OF DYNAMIC EQUATIONS OF NON-NEWTON FLUID

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1. In this article, with the use of approximate group analysis developed in [1]–[3], we study the dynamic equation of plane surface of non-Newton fluid subordinate to the Reiner–Rivlin rheological law. It is convenient to write this equation in the form (see [4])

$$u_t = (u^2\Phi(\sigma))_x + \varepsilon F(t, x, u, \varepsilon), \quad \sigma = uu_x. \tag{1.1}$$

Here t is the time, x is the longitudinal coordinate; the unknown function $u(t, x)$ determines the plane surface of fluid, and Φ is an arbitrary function determined by the fluid rheology (usually, in applications, Φ is an odd monotone increasing continuous function), εF is the mass balance of fluid (a near-surface source), ε is a small parameter.

If $\varepsilon = 0$, the unperturbed equation

$$u_t = (u^2\Phi(\sigma))_x, \quad \sigma = uu_x, \tag{1.2}$$

admits the group generated by translations along t and x whose infinitesimal operators are $X_1^0 = \partial_t$ and $X_2^0 = \partial_x$, respectively, and the scaling group whose operator is $X_3^0 = t\partial_t + 2x\partial_x + u\partial_u$.

Note that in [5] the approximate symmetries of (1.1) with Φ being a power function were completely studied. This corresponds to the Ostwald–Lie rheological law for non-Newton fluid (a partial case of the general Reiner–Rivlin rheological law). Also in [5] approximately invariant solutions of the obtained approximately invariant equations were constructed. In [6], [7], for a power function Φ, conditions under which the approximate symmetries of (1.1) are exact, were considered in detail and exact invariant solutions were constructed. In this article we widely use the technique developed in [5]–[7].

2. We shall use results of [1]–[3], therefore the following notation is convenient:

$$\begin{matrix} t & x & u & \sigma & u_t & u_x & \sigma_t & \sigma_x \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8. \end{matrix}$$

With this notation, (1.1) is written as

$$F_0^1 + \varepsilon F_1^1 = 0, \quad F_0^2 + \varepsilon F_1^2 = 0, \tag{2.1}$$

where

$$F_0^1 = \frac{\partial}{\partial y_2}(y_3^2\Phi(y_4)) - y_5, \quad F_1^1 = F(y_1, y_2, y_3, y_4), \quad F_0^2 = y_4 - y_3 y_6, \quad F_1^2 = 0.$$

By [1], an infinitesimal operator of one-parameter group of approximate up to $O(\varepsilon^2)$ symmetries of (1.1) is generated by an operator of one-parameter group of exact symmetries of unperturbed

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