ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2012, Vol. 56, No. 4, pp. 44–53. © Allerton Press, Inc., 2012.
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# Formation of Nonsmoothness Lines in Optimization of Screw Pairs in Screw Pumps

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Received March 22, 2011

**Abstract**—Computer modeling of screw pairs in screw pumps reveals the formation of nonsmoothness lines on the screw surface (with the growth of the height of the helical tooth) conjugate to the smooth helical surface. The mathematical explanation of the appearance of such lines represents considerable interest for the theory and practice of pumps manufacturing. In this paper we show that one can explain the mentioned phenomenon by studying a wider problem, namely, the bifurcation of regressive points on a plane contour conjugate to the smooth one.

**DOI:** 10.3103/S1066369X12040068

Keywords and phrases: *screw pumps, screw pairs, conjugate contour, points of nonsmoothness, bifurcation of regressive points, computer identification of singularities.*

**Introduction. The problem.** The problem of mathematical modeling and optimization of gear screw pumps is a subproblem of the general problem of projection and optimization of multiphase screw pumps [1–3]. We study kinematic and geometric properties of screw systems by analyzing one-parameter periodic families of smooth planar contours (lateral sections of screws). This allows us to apply the theory of smooth maps [4, 5] of two-dimensional tori into the coordinate plane.

In this paper we consider the case when screw pairs have the same number of starts, but one can easily extend the obtained results for the case of multiple-start screw surfaces[1)] (including the case of $n$-screw systems, $n > 2$, with various numbers of starts).

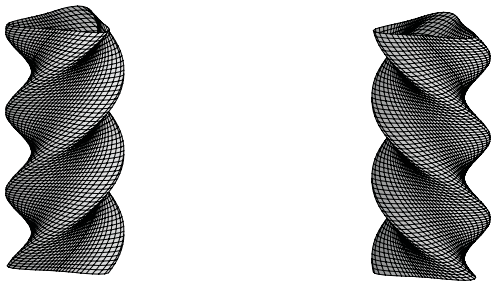

**Fig. 1.** A (non-optimized) pair of 3-start screw surfaces.

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[1)] If $\omega$, $0 < \omega < 2\pi$, is the (least) period of a profile function $f(\varphi)$, then $\omega = \frac{2\pi}{n}$, where $n$ is an integer (positive) number; it is called the *number of starts* of the screw (the $n$-start screw). The number of starts coincides with the number of teeth of the screw in its lateral section. The difference between the (absolute) maximum and minimum of $f$ is called the height of the tooth in the lateral section of the screw.