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The Structure of Dendrites with the Periodic Point Property

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Abstract—In this paper we study the structure of dendrites with the periodic point property, i.e., dendrites X such that for any continuous map $f : X \to X$ and any subcontinuum $Y \subset X$ the condition $Y \subset f(Y)$ implies that Y contains a periodic point of f.

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The question on the existence of periodic points represents one of the main problems in the theory of dynamic systems. This problem was studied for continuous maps on dendrites, for example, in [1–4]. In [1] one has proved the existence theorem for a fixed point of a continuous map of a dendrite to itself. In [2] one studied the existence conditions for a fixed point for homeomorphisms and monotone surjections of a dendrite X on a closed subset $M \subset X$. In [3] one has obtained the existence conditions for a periodic point of a continuous map of a dendrite X on some connected component $U \subset X$. In [4] one has proved the existence of a periodic point of a continuous map $f : X \to X$ of a dendrite X on a subcontinuum $Y \subset X$ under condition that $Y \subset f(Y)$ and Y is a finite tree. One has shown that the indicated result cannot be generalized to the case when Y is a dendrite with a countable set of branch points.

In this paper we study the structure of dendrites that have the periodic point property.

The interest to studying dynamic systems on dendrites is connected, for example, with the fact that dendrites arise as Julia sets in complex dynamic systems ([5], P. 14). On the other hand, dendrites present examples of Peano continua with a complex topological structure ([6], pp. 165–187).

1. PRELIMINARY INFORMATION

A dendrite is a locally connected continuum (*a compact connected metric space*) *which contains no subsets homeomorphic to a circumference.*

We understand an arc in a dendrite X as a set homeomorphic to a closed interval on the straight line (we treat an one-point set as a degenerate arc). One can connect any two points x and y, $x \neq y$, in a dendrite by a unique arc with endpoints x and y. We denote by the symbol $[x, y]$ the arc with endpoints x and y including these points; we set $(x, y) = [x, y] \setminus \{x, y\}$.

In this paper we use the definition of the order of a point in the Menger–Urysohn sense (for example, [7], § 51).

Let X be a dendrite and let n be a cardinal number not greater than c or an ordinal number ω of the set of all nonnegative numbers in their natural order. We say that the order of a point $x \in X$ does not exceed n ($\mathrm{ord}_X\, x \leq n$) *if for any $\varepsilon > 0$ there exists $0 < \delta < \varepsilon$ such that* $\mathrm{card}(\partial U_\delta(x)) \leq n$, *where $\partial U_\delta(x)$ is the boundary of the δ-neighborhood of the point x, and* $\mathrm{card}(\cdot)$ *is the power of the set* $(\cdot)$. *The equality* $\mathrm{ord}_X\, x = n$ *means that* $\mathrm{ord}_X\, x \leq n$ *and the correlation* $\mathrm{ord}_X\, x \leq m$ *is false with any $m < n$.*

The points whose order exceeds two are called the branch points. The points whose order equals one are called endpoints. We denote the set of branch points (endpoints) of a dendrite X by $R(X)$

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