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Time Localization of Alpert Multiwavelets

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Abstract—We study the behavior of radii of Alpert multiscaling functions of arbitrary dimensions. We calculate the radii up to the 4th order for the corresponding multiwavelets. In addition, we obtain an integral correlation for the Legendre polynomials.

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One of important characteristics of a wavelet in the scalar case is its frequency-time localization or the constant of uncertainty. The constants of uncertainty for multiwavelets are studied in the paper [1], where the radii are calculated in terms of multiscaling coefficients.

In this paper we evaluate the radii for multiscaling functions and Alpert multiwavelets in a time domain without the use of multiscaling coefficients.

Let us first introduce some necessary notions and then formulate the main result.

Definition 1. A vector function

$$\boldsymbol{\varphi} = (\varphi_0, \varphi_1, \dots, \varphi_{r-1})^\top,$$

where functions

$$\varphi_0, \varphi_1, \dots, \varphi_{r-1} \in L_1(\mathbb{R}) \bigcap L_2(\mathbb{R})$$

have compact supports, is said to be multiscaling, if it satisfies either correlations

$$\varphi_l(t) = \sqrt{2} \sum_{k\in\mathbb{Z}} h_{l0}^{(k)} \varphi_0(2t-k) + h_{l1}^{(k)} \varphi_1(2t-k) + \dots + h_{l(r-1)}^{(k)} \varphi_{(r-1)}(2t-k)$$

for each $l = 0, \dots, r-1$ or the so-called multiscaling equation

$$\boldsymbol{\varphi}(t) = \sqrt{2} \sum_{k\in\mathbb{Z}} H_k \boldsymbol{\varphi}(2t-k),$$

where

$$H_k = \begin{pmatrix} h_{00}^{(k)} & h_{01}^{(k)} & \dots & h_{0(r-1)}^{(k)} \\ h_{10}^{(k)} & h_{11}^{(k)} & \dots & h_{1(r-1)}^{(k)} \\ \dots & \dots & \dots & \dots \\ h_{(r-1)0}^{(k)} & h_{(r-1)1}^{(k)} & \dots & h_{(r-1)(r-1)}^{(k)} \end{pmatrix}$$

are coefficients of the multiscaling equation.

A multiscaling vector function φ is called orthogonal, if

$$\int_{\mathbb{R}} \varphi(x)\varphi^*(x-k)dx = \delta_{0k} I.$$

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