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Generalized Elements in Problem on Asymptotic Attainability

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Abstract—We consider a problem which implies the choice of a solution subject to asymptotic constraints. We represent the results as ultrafilters of the space of ordinary estimates (the space is not necessarily endowed with a topology). This representation corresponds to an abstract attainability problem in its nonsequential asymptotic version.

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1. INTRODUCTION

In this paper we consider a problem connected with the choice of a solution under rather general constraints. Each solution is associated with an element of a certain given set; we call this element an estimate. An abstract problem considered below has a prototype. The latter is the well-known in the control theory problem which implies the construction and the study of the attainability domain. Relaxation of a system of constraints may cause a sudden expansion of the domain (considered as a set). Note that under "small" perturbations the points of a new attainability domain (a wider one) actually characterize the control abilities. As a rule, it is impossible to evaluate the measure of smallness directly; however, one can "bypass" this difficulty, using approximate (in fact, asymptotic) solutions calculated in a way similar to that of J. Varga ([1], Chaps. III, IV). Following [1], one can use the so-called generalized controls in order to define the real abilities with respect to the attainability of terminal states.

The mentioned generalized controls were used by N. N. Krasovskii and his followers in the solution of game problems in dynamics [2, 3] In particular, the use of generalized elements is essential in the definition of the set stability [2] proposed by N. N. Krasovskii. Using this important property and the method of extremal shift, N. N. Krasovskii and A. I. Subbotin established the fundamental theorem on an alternative in a nonlinear differential game. Here the generalized controls (zero-overshoot responses) also properly characterize the corresponding asymptotic behavior of ordinary controls.

In the mentioned cases, as well as in several other ones, an approximate validity of usual constraints (rather than the strict one) allows us to obtain some new results which are valuable both for the theory and practice. At the same time, restricting ourselves with the use of only the sequences of ordinary solutions as variants of the asymptotic behavior, we can encounter certain difficulties. This does not occur in the usual control problems with geometric constraints (they were first systematically studied by L. S. Pontryagin). In particular, this is true for problems connected with the construction of infinite-dimensional analogs of attainability domains in topological spaces. Consider, for example, the construction of an asymptotic version of a bunch of trajectories, when the corresponding space of functions is endowed with the topology of pointwise convergence.

In this connection, we do not restrict ourselves with the use of sequences as "asymptotic" solutions, but also consider directions and filters in the space of ordinary solutions (this appears to be even more convenient). Note that filters and directions may be useful even in those cases, when the sequential versions of a solution similar to that constructed by J. Varga enable one to obtain all attainability elements. The reason is that constructing a certain sequence of ordinary solutions, one can meet a situation, when this sequence has to be "extracted" with the help of the countable choice axiom. At the

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