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Multivalued Functions and Nonwandering Set of Skew Products of Maps of an Interval With Complicated Dynamics of Quotient Map

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Abstract—With the use of special multivalued functions, we give a description of nonwandering set of C^1-smooth skew products of maps of an interval with Ω-stable quotient map of the type $\succ 2^\infty$.

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1. INTRODUCTION

In this article, we give a description of a nonwandering set (see the definition in [1]) for some C^1-smooth skew products of interval maps with Ω-stable quotient map (quotient) with complicated dynamics. In modern mathematical literature, there are only papers dedicated to various aspects of the structure of nonwandering set for skew products of interval maps whose quotients have simple dynamics, i.e., skew products with either closed set of periodic points [2, 3], or with closed set of periodic points of quotients [4–8] (at that the set of periodic points of skew product can be non-closed).

Let $I = I_1 \times I_2$ be a closed rectangle on the plane (I_1 and I_2 are segments). Consider skew product of interval maps, i.e., a dynamical system $F : I \to I$ of the form

$$F(x, y) = (f(x), g_x(y)), \text{ where } g_x(y) = g(x, y), \ (x; y) \in I. \tag{1}$$

Here $f : I_1 \to I_1$ is a *quotient* of a skew product (1), and for every $x \in I_1$, $g_x : I_2 \to I_2$ is a map, *acting in the fiber over point* x.

Due to equality (1), for all $n > 1$ we get

$$F^n(x, y) = (f^n(x), g_{x,n}(y)), \text{ where } g_{x,n} = g_{f^{n-1}(x)} \circ \cdots \circ g_x. \tag{2}$$

Henceforward, we will use the notation $\widetilde{g}_x$ for map $g_{x,n}$, if x is a periodic point of f ($x \in \mathrm{Per}(f)$), and n is its (least) period.

Denote by $T^0(I)$ ($T^1(I)$) the space of all continuous (C^1-smooth) skew products of interval maps endowed with a standard C^0-norm (C^1-norm).

Let $C^1_{\partial_k}(I_k)$ ($k = 1, 2$) be a subspace of the space $C^1(I_k)$ of C^1-smooth interval maps $\psi \in C^1(I_k)$ on the segment I_k, satisfying the condition of ψ-invariance of the segment's border ∂I_k: $\psi(\partial I_k) \subset \partial I_k$.

Denote by $C^1_\omega(I_k)$ the space of all Ω-stable in $C^1_{\partial_k}(I_k)$ maps of the segment I_k into itself ($k = 1, 2$).

Following [9], let us formulate the basic properties of Ω-stable interval maps.

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