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Attractors of Weak Solutions to a Regularized System of Motion Equations for Fluids with Memory

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Abstract—In this paper we announce the existence of trajectory and global attractors for weak solutions to a regularized model of flows with memory.

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1. INTRODUCTION

The solvability in the weak sense of the boundary-value problem for a regularized model of flows with memory has been established in papers [1, 2] and described in detail in [3].

Let us consider the boundary-value problem in a bounded domain $\Omega \subset \mathbb{R}^n$ with the boundary Γ ($n = 2, 3$) that corresponds to a regularized model of the motion of a fluid with memory:

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \mu_1 \operatorname{Div} \int_0^t e^{-\frac{t-s}{\lambda}} \mathcal{E}(v)(s, Z_\delta(s; t, x))\, ds - \mu_0 \operatorname{Div} \mathcal{E}(v)$$
$$= -\operatorname{grad} p + \varphi, \quad (t, x) \in (0, T) \times \Omega, \quad (1)$$

$$\operatorname{div} v = 0, \quad (t, x) \in (0, T) \times \Omega; \quad v\big|_{(0,T)\times\Gamma} = 0, \quad (2)$$

$$v(0, x) = v^0(x), \quad x \in \Omega; \quad \int_\Omega p\, dx = 0, \quad (3)$$

where $v = (v_1, \dots, v_n)$ is the vector function of the motion rate of medium particles, p is the medium pressure, φ is the density of applied forces, $\mathcal{E} = (\mathcal{E}_{ij})$ is the tensor of deformation rates whose components are defined by formulas $\mathcal{E}_{ij} = 1/2(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$ $(i, j = 1, \dots, n)$; $\operatorname{Div} a$ is the divergence of the matrix $a = (a_{ij})$ of the order n, i.e., $\operatorname{Div} a = \Big(\sum\limits_{i=1}^{n} \frac{\partial a_{i1}}{\partial x_i}, \dots, \sum\limits_{i=1}^{n} \frac{\partial a_{in}}{\partial x_i} \Big)$, and $\mu_1 \geqslant 0$ is a real number.

Let us describe $Z_\delta(s; t, x)$. To this end we introduce the standard (in mathematical problems of hydrodynamics) spaces V and H.

We denote by $\mathfrak{D}(\Omega)^n$ the space of functions of the class C^∞ such that they are defined on Ω, take on values in $\mathbb{R}^n$, and their compact carriers belong to Ω. Let $\mathcal{V} = \{v : v \in \mathfrak{D}(\Omega)^n, \operatorname{div} v = 0\}$ be the subset of solenoidal functions in $\mathfrak{D}(\Omega)^n$. We denote by the symbol H the closure of $\mathcal{V}$ with respect to the norm of the space $L_2(\Omega)^n$, and we do by V be the closure of $\mathcal{V}$ with respect to the norm of the space $W_2^1(\Omega)^n$.

Consider the trajectory defined by the equation

$$z(\tau; t, x) = x + \int_t^\tau S_\delta v(s, z(s; t, x))\, ds, \quad \tau \in [0, T], \ (t, x) \in (0, T) \times \Omega. \quad (4)$$

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