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The Vallée-Poussin Means for Special Series With Respect to Ultraspherical Jacobi Polynomials With Sticking Partial Sums

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Abstract—The authors continue study of special series with sticking property (r-fold coincidence at points ± 1) in ultraspherical Jacobi polynomials, that was started in the previous works of the first author. In the present paper they are dealing with an approximative properties of Vallée-Poussin means for partial sums of the mentioned special series. It is shown that for function f with certain smoothness properties at the ends of interval $[-1,1]$ the rate of weighted approximation by Vallée-Poussin means has the same order as the best weighted approximation of f.

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1. INTRODUCTION

Let a function f be defined on segment $[-1,1]$ and have derivatives $f^{(\nu)}(\pm 1)$, $\nu = 0,1,\dots,r-1$. Then we can define the Hermite interpolation polynomial for this function

$$D_{2r-1}(f,x) = \frac{(1-x^2)^r}{2^r}\sum_{\nu=0}^{r-1}\frac{1}{\nu!}\sum_{s=0}^{r-1-\nu}\frac{(r)_s}{2^s s!}\left[\frac{f^{(\nu)}(-1)}{(1+x)^{r-\nu-s}} + \frac{(-1)^\nu f^{(\nu)}(1)}{(1-x)^{r-\nu-s}}\right],$$

which coincides with f at points ± 1 with multiplicity r. Let $C_r[-1,1]$, $r \geq 1$, stand for the space of continuous on $[-1,1]$ functions f such that there exist derivatives $f^{(\nu)}(\pm 1)$ for $\nu = 0,\dots,r-1$, and

$$\mathcal{E}_r(f) = \sup_{-1<x<1}\frac{|f(x) - D_{2r-1}(f,x)|}{(1-x^2)^{\frac{r}{2}}} < \infty. \tag{1.1}$$

If $f \in C_r[-1,1]$, then we consider the error of the best weight approximation

$$E_n^r(f) = \inf_{q_n \in Q^{n,r}(f)}\sup_{-1<x<1}\frac{|f(x) - q_n(x)|}{(1-x^2)^{\frac{r}{2}}},$$

where $Q^{n,r}(f)$, $n \geq 2r-1$, is set of all algebraic polynomials q_n of degree n such that $q_n^{(\nu)}(\pm 1) = f^{(\nu)}(\pm 1)$ for $\nu = 0,\dots,r-1$. Note that by virtue of (1.1) there is valid the inequality $E_n^r(f) \leq \mathcal{E}_r(f)$. In addition, if the function $f(x)$ has r continuous derivatives on segment $[-1,1]$, then by Telyakovskii–Gopengauz theorem [1, 2] we have $E_n^r(f) \leq cn^{-r}\omega(f^{(r)}, 1/n)$, where $\omega(f,x)$ is modulus of continuity for function $f \in C[-1,1]$; here and in what follows $c, c(\alpha), \dots$ stand for positive values depending only on the written parameters.

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