Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 45, No. 5, pp. 69–77, 2001 *UDC 512.541*

U-SEQUENCES AND ALMOST ISOMORPHISM BETWEEN THE ABELIAN p-GROUPS WITH RESPECT TO FULLY INVARIANT SUBGROUPS

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Two groups, each of them being isomorphic to a subgroup of another group, are said to be *almost isomorphic* (see [1]). Two groups are said to be *almost isomorphic with respect to subgroups with a certain property* if each of them is being isomorphic to a subgroup of the other group, which possesses such a property. In this article we study the problem of isomorphism between the Abelian p-groups of certain classes, which are almost isomorphic with respect to fully invariant subgroups. Our objective is to give an answer to the question: Whether an analog of the well-known set-theoretic Cantor–Schröder–Bernstein theorem in the situation under consideration can be true or not?

This problem is solved in the class of reduced separable p-groups almost isomorphic with respect to fully invariant subgroups. We do not consider non-reduced p-groups, since an almost isomorphism between the two Abelian groups with respect to fully invariant subgroups implies an isomorphism between these groups if and only if an almost isomorphism between their reduced parts implies an isomorphism between these parts (see [2]).

Recall that in accordance with I. Kaplansky (see [3], p. 61) any fully invariant subgroup of a reduced separable p-group G is uniquely determined by some U-sequence α for G. We will denote by $G(\alpha)$ this fully invariant subgroup.

Definition. We say that a pair of sequences (α, β) *specifies an almost isomorphism* with respect to fully invariant subgroups in the class of reduced separable p-groups if in this class groups G and G' exist such that α and β are U-sequences for G' and G, respectively, and $G \cong G'(\alpha)$, $G' \cong G(\beta)$.

The following two questions arise in a natural way: 1) *Which pairs of sequences specify an almost isomorphism with respect to fully invariant subgroups in the class of reduced separable p-groups*? 2) *Does this almost isomorphism imply an isomorphism between the groups*?

The objective of this article is to resolve this problem.

1. U-sequences

We will use the following notation and facts.

Let G be a reduced separable p-group and σ an ordinal. We denote by $p^\sigma G$ the subgroup of G which is defined inductively as follows: $p^0 G = G$, $p^{\sigma+1}G = p(p^\sigma G)$, and $p^\sigma G = \bigcap_{\rho<\sigma} p^\rho G$ if σ is the limit ordinal. The least ordinal σ such that $p^{\sigma+1}G = p^\sigma G$ is called *the length* of G (see [4], p. 181). The *σ-th Ulm–Kaplansky invariant* of G is the following cardinal (ibid., p. 182)

$$f_G(\sigma) = r((p^\sigma G)[p]/(p^{\sigma+1}G)[p]).$$

In addition, we denote by Z_0 the set of nonnegative integers.