Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 46, No. 5, pp. 42–45, 2002 *UDC 512.579*

ELEMENTARY EQUIVALENCE OF DERIVATIVE STRUCTURES OF FREE LATTICES

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In the study of free algebras, properties expressible in the first order language play an important role. Since the number k of free generators of a free algebra $\mathcal{F}_V(k)$ from a variety V is an invariant, the elementary equivalence of free algebras $\mathcal{F}_V(k) \equiv \mathcal{F}_V(\lambda)$ induces an equivalence on the class Card of all cardinals. As is well-known, this equivalence is trivial on the class of all infinite cardinals: $\mathcal{F}_V(k) \equiv \mathcal{F}_V(\lambda)$ for any $k, \lambda \geq \aleph_0$. In this connection, it is natural to use in the framework of the above indicated theory instead of free algebras $\mathcal{F}_V(k)$ some derivative structures such as: lattices of subalgebras $\operatorname{Sub}\mathcal{F}_V(k)$, lattices of congruences $\operatorname{Con}\mathcal{F}_V(k)$, automorphism groups $\operatorname{Aut}\mathcal{F}_V(k)$, semigroups $\operatorname{Iso}\mathcal{F}_V(k)$ of inner isomorphisms (isomorphisms between subalgebras of $\mathcal{F}_V(k)$), endomorphism semigroups $\operatorname{End}\mathcal{F}_V(k)$, semigroups $\operatorname{Ihm}\mathcal{F}_V(k)$ of inner homomorphisms (homomorphisms between subalgebras of $\mathcal{F}_V(k)$), and others. Equivalencies on the class Card which arise in these cases rather often turn out to be equivalencies of cardinals in a certain second order logic. For a survey of related results see [1]. In this article similar results are obtained for the varieties of lattices.

Recall the well-known result on second order logics by S. Shelah (see [2]). In the statement of the theorem it is assumed that a collection of relations over which quantification is admitted is singled out from the collection of all relations by a first order formula.

Theorem A (S. Shelah). *Only four second order logics exist* (*up to mutual interpretability of second order logics in each other*):

1) *the first order logic* (*quantification over relations is not admissible*);
2) *the monadic second order logic* (*quantification is admissible over any unary relations*);
3) *permutational second order logic* (*quantification is admissible over relations which are bijections on the underlying set of the model*);
4) *complete second order logic* (*quantification is admissible over any relations*).

Equivalence of the two models $\mathcal{A}$ and $\mathcal{B}$ in the first order logic will be denoted by $\mathcal{A} \equiv \mathcal{B}$, in the permutational logic by $\mathcal{A} \equiv_p \mathcal{B}$, and in the complete second order logic by $\mathcal{A} \equiv_2 \mathcal{B}$. Speaking about equivalence of cardinals k and λ in a second order logic, we mean equivalence of these cardinals as models of empty signature. As usual, by a cardinal k we will mean the collection of all cardinals which are less than k. The elements of the cardinal k are assumed to be the generators of the algebra $\mathcal{F}_V(k)$.

Theorem 1. *For any nontrivial finitely basable variety of lattices V, for any infinite cardinals k and λ, we have*

1) $\operatorname{Sub}\mathcal{F}_V(k) \equiv \operatorname{Sub}\mathcal{F}_V(\lambda) \iff k \equiv_2 \lambda$;
2) $\operatorname{Aut}\mathcal{F}_V(k) \equiv \operatorname{Aut}\mathcal{F}_V(\lambda) \iff k \equiv_p \lambda$;

Supported by the Russian Foundation for Basic Research (grant no. 99-01-00571).