ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2012, Vol. 56, No. 8, pp. 35–43. © Allerton Press, Inc., 2012.
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A Problem with the Frankl and Bitsadze–Samarskii Condition on the Line of Degeneracy and on Parallel Characteristics for a Mixed-Type Equation

M. Kh. Ruziev[1*]

[1] *Institute of Mathematics and Information Technologies, National Academy of Sciences of Republic of Uzbekistan, ul. Durman yuli 29, Tashkent, 100125 Republic of Uzbekistan*

Received March 31, 2011

Abstract—In this paper we study the boundary-value problem with the Frankl and Bitsadze–Samarskii condition on the line of degeneracy and on parallel characteristics for a mixed-type equation with a singular coefficient. We prove the existence of a solution by the method of integral equations, and we do its uniqueness with the help of the extremum principle.

DOI: 10.3103/S1066369X12080051

Keywords and phrases: *extremum principle, unique solvability, existence of solution, integral equations.*

1. INTRODUCTION AND PROBLEM DEFINITION

Let $D = D^+ \cup D^- \cup I$ be a domain in the complex plane $z = x + iy$, where D^+ is the half-plane $y > 0$, and D^- is a finite domain of the half-plane $y < 0$ bounded by characteristics AC and BC of the equation

$$(\operatorname{sign} y)|y|^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = 0, \tag{1}$$

issuing from points $A(-1, 0)$ and $B(1, 0)$, and the segment AB of the straight line $y = 0$, $I = \{(x, y) : -1 < x < 1,\ y = 0\}$. We assume that m, β_0 in Eq. (1) are some real numbers satisfying the conditions $m > 0$ and $-m/2 < \beta_0 < 1$.

Let D_R^+ be a finite domain cut out of the domain D^+ by the arc $A_R B_R$ of the normal curve

$$x^2 + 4y^{m+2}/(m+2)^2 = R^2,\ \ -R \le x \le R,\ \ 0 \le y \le ((m+2)R/2)^{2/(m+2)},\ \ A_R(-R, 0),\ B_R(R, 0).$$

Introduce the following denotations: $I_1 = \{(x, y) : -\infty < x < -1,\ y = 0\}$, $I_2 = \{(x, y) : 1 < x < \infty,\ y = 0\}$, C_0 (C_1) are points of the intersection of the characteristic AC (BC) with that issuing from the point $E(c, 0)$, where $c \in I$ is an arbitrary fixed number, $D_R = D_R^+ \cup D^-$, D_R is a subdomain of the unbounded domain D.

Consider a diffeomorphism $q(x) \in C^1[c, 1]$ that maps a segment $[c, 1]$ to that $[-1, c]$, while $q'(x) < 0$, $q(c) = c$, and $q(1) = -1$. An example of such a function is $q(x) = p - kx$, where $k = (1 + c)/(1 - c)$, $p = 2c/(1 - c)$, $p - k = -1$, and $p - kc = c$.

In the paper [1] in a bounded domain one studies a problem, where the characteristic AC is arbitrarily split into two pieces (AC_0, C_0C). On the first piece one states the Tricomi condition ([2], P. 29), and on the second piece and on its parallel characteristic one does the Bitsadze–Samarskii condition [3].

In this paper we study the problem in an unbounded area. Unlike [1], the characteristic AC_0 is free of the edge condition (the Tricomi condition); it is equivalently replaced with the Frankl nonlocal condition [4–10] on a segment of the degeneracy line.

*E-mail: mruziev@mail.ru.