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SUFFICIENT CONDITIONS FOR STABILITY OF LINEAR STOCHASTIC SYSTEM WITH AFTEREFFECT WITH RESPECT TO A PART OF VARIABLES

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Let $(\Omega, \mathcal{F}, (\mathcal{F})_{t\geq 0}, P)$ be a stochastic basis (see [1], p. 9), $\bigvee_{s=0}^{t} g(s)$ stand for total variation of a function $g(s)$ on the segment $[0,t]$, D^n a linear space of n-dimensional predictable (ibid., p. 13) random processes on $[0,+\infty[$ such that their trajectories are almost surely (a. s.) continuous from the right and possess a limit from the left, k^n a linear space of n-dimensional $\mathcal{F}_0$-measurable random values; $Z = \mathrm{col}(z^1, \dots, z^m)$ an m-dimensional semimartingale (ibid., p. 73); $L^n(Z)$ a linear space of predictable $n \times m$-matrices on $[0,+\infty[$ with rows locally integrable in the semimartingale Z; $\lambda : [0,+\infty[\to R_+$ a non-decreasing function; λ^* a measure generated by the function λ; L_q^λ a linear space of scalar functions on $[0,+\infty[$, summable with power q for $1 \leq q < \infty$ in the measure λ^* and essentially bounded for $q = \infty$ in the same measure λ^*; R^n a linear space of n-dimensional vectors with the norm $|\cdot|$; $\|\cdot\|$ the norm of $l \times n$-matrix, coordinated with the norm $|\cdot|$; $\|\cdot\|_X$ stands for the norm of the normed space X; $\|x\|_t = \sup_{0\leq s\leq t} |x(s)|$ if $x \in R^n$, and $\|H\|_t = \sup_{0\leq s\leq t} \|H\|$ if H is $l \times n$-matrix. Assume also that $1 \leq p < \infty$, and c_p is a positive constant depending on p from inequality (9.48) in [1] (see p. 65 there).

In [2], for an arbitrarily fixed value p and a positive scalar function $\gamma(t)$, $t \in [0,+\infty[$, the normed spaces M_p^γ ($M_p^1 = M_p$), k_p^n were introduced. For any $x \in D^n$, $x = \mathrm{col}(x^1, \dots, x^n)$, $0 < k < n$, we introduce the following notation: $y = \mathrm{col}(x^1, \dots, x^k)$, $h = \mathrm{col}(x^{k+1}, \dots, x^n)$. Then $x = \mathrm{col}(y, h)$ and $D^n = D^k \times D^{n-k}$.

In what follows we suppose that the semimartingale Z is representable as $Z = b + c$, where b is a predictable random process of locally bounded variation and c is a locally square integrable martingale (see [1], p. 28). In addition, all the components of the process b and the mutual characteristics $\langle c^i, c^j\rangle$ (ibid., p. 48) of all components of the martingale c are assumed to be absolutely continuous with respect to the measure λ^*. The latter means that

$$b^i = \int_0^{\cdot} a^i d\lambda, \quad \langle c^i, c^j\rangle = \int_0^{\cdot} A^{ij} d\lambda \quad (i,j = 1, \dots, m).$$

Let $a = \mathrm{col}(a^1, \dots, a^m)$; $A = [A^{ij}]$ be an $n \times m$-matrix; $a^+ = \mathrm{col}(|a^1|, \dots, |a^m|)$; $A^+ = [|A^{ij}|]$. If B is an $n\times n$-matrix, then B^1 stands for a $k\times k$-matrix obtained from the matrix B via the deletion of its last $n-k$ columns and rows, B^2 stands for a $k \times (n-k)$-matrix obtained from the matrix B via the deletion of its first k columns and $n-k$ last rows, B^3 stands for an $(n-k) \times k$-matrix obtained from the matrix B via the deletion of its first k rows and last $n-k$ columns, and B^4 is an $(n-k) \times (n-k)$-matrix obtained from the matrix B via the deletion of first k both rows and columns.
