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STABILITY OF SOLUTIONS OF STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS WITH RESPECT TO PART OF VARIABLES BY FIRST APPROXIMATION

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Basing on the approach exposed in [1], in this article we investigate the p-stability ($1 \le p < \infty$) with respect to a part of variables of the solutions of nonlinear stochastic functional differential equations by the semimartingale.

Let $(\Omega, \mathcal{F}, (\mathcal{F})_{t\ge 0}, P)$ be the stochastic basis (see [2], p. 9); D^n a linear space of n-dimensional predictable (see [2], p. 13) random processes on $[0, +\infty[$, whose trajectories are almost surely (a. s.) continuous from the right and possess the limit from the left; k^n a linear space of n-dimensional $\mathcal{F}_0$-measurable random values; $Z = \mathrm{col}(z^1, \dots, z^m)$ the m-dimensional semimartingale (see [2], p. 73); $L^n(Z)$ a linear space of predictable $n \times m$-matrices on $[0, +\infty[$, whose rows are locally integrable by the semimartingale Z (see [3]); $\lambda : [0, +\infty[\to R_+$ a certain nondecreasing function; λ^* a measure generated by the function λ; L_p^λ a linear space of scalar functions on $[0, +\infty[$, integrable with power q for $1 \le q < \infty$ with respect to the measure λ^* and bounded in essential for $q = \infty$ by the measure λ^* ($L_q^t = L_q$); R^n a linear space of n-dimensional vectors with the norm $|\cdot|$; $\|\cdot\|$ the norm of an $l \times n$-matrix, coordinated with the matrix $|\cdot|$; $\|\cdot\|_X$ the norm in the normed space X; E the mathematical expectation; $1 \le p < \infty$; $1 \le q \le \infty$; $1 \le k < n$, $\gamma_i : [0, +\infty[\to R^1$ a positive function for $i = 1, 2$; $\xi : [0, +\infty[\to R^1$ a nonnegative function locally integrable by the measure λ^*; B a linear normed subspace of the space $L^n(Z)$.

In what follows the semimartingale Z is assumed to possess a. s. continuous trajectories and to be representable in the form $Z = b + c$, where b is a predictable random process of locally bounded variation, while c is a locally quadratic integrable martingale (see [2], p. 28). In addition, all components of the process b and mutual characteristics $\langle c^i, c^j \rangle$ (see [2], p. 48) of all the complements of the martingale c are assumed to be absolutely continuous with respect to the measure λ^*. The latter means that

$$b^i = \int_0^{\cdot} a^i d\lambda, \quad \langle c^i, c^j \rangle = \int_0^{\cdot} A^{ij} d\lambda \quad (i, j = 1, \dots, m).$$

Let $a = \mathrm{col}(a^1, \dots, a^m)$, $A = [A^{ij}]$ be an $n \times m$-matrix. As is known (see [3]), in this case the space $L^n(Z)$ consists of predictable $n \times m$-matrices H, for which the inequality is fulfilled: $\int_0^t (|Ha| + \|HAH^\top\|) d\lambda < \infty$ a.s. for any $t \ge 0$ and $\int_0^t H dZ = \int_0^t H db + \int_0^t H dc$. In addition, the inequality takes place: $\Big(E\Big|\int_0^t H dZ\Big|^{2p}\Big)^{1/2p} \le \Big(E\Big(\int_0^t |Ha| d\lambda\Big)^{2p}\Big)^{1/2p} + c_p\Big(E\Big(\int_0^t \|HAH^\top\| d\lambda\Big)^p\Big)^{1/2p}$, where c_p is a positive number depending on p (see [2], p. 65).

For any $x \in D^n$ and $f \in L^n(Z)$, we have $y = \mathrm{col}(x^1, \dots, x^k)$, $h = \mathrm{col}(x^{k+1}, \dots, x^n)$, $f^y = \mathrm{col}(f^1, \dots, f^k)$, $f^h = \mathrm{col}(f^{k+1}, \dots, f^n)$ ($f^y \in L^k(Z)$, $f^h \in L^{n-k}(Z)$, $f = \mathrm{col}(f^y, f^h)$). We introduce the following normed spaces:

$$k_p^n = \{\alpha : \alpha \in k^n, \ E|\alpha|^p < \infty\}$$