ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2015, Vol. 59, No. 12, pp. 10–20. © Allerton Press, Inc., 2015.
Original Russian Text © V.P. Derevenskii, 2015, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2015, No. 12, pp. 14–26.

Lagrange Matrix Equations

V. P. Derevenskii[1*]

[1] *Kazan State Architecture and Building University, ul. Zelyonaya 1, Kazan, 420043 Russia*

Received April 17, 2014

Abstract—In this paper, we proceed with studying matrix equations over "skew series". We establish conditions for splitting a Lagrange matrix equation into a set of scalar differential equations. We consider diagonal, triangular, nil-triangular, and dual-diagonal forms of its solution.

DOI: 10.3103/S1066369X15120026

Keywords: *ordinary differential equations, matrices, skew series.*

1. The problem. Let us introduce the necessary denotations. The symbol M_n stands for the set of square $n \times n$ matrices over the real number field R with the unit E and basic elements of the full matrix algebra $E_{ij} : i, j = \overline{1, n}$; t is a continuous independent variable in R; $\Phi^k(t)$ is the set of functions from R to R which are k times continuously differentiable in t; $M_n^k(t)$ is the set of matrices from M_n, whose elements belong to $\Phi^k(t)$; and N is the set of positive integers.

Consider the following Lagrange matrix equation (LME):

$$X = tF(X') + G(X') : X \in M_n^1(t), \;\; X' \equiv \frac{dX}{dt}, \tag{1}$$

$$F(X') \equiv \sum_{m=0}^{\infty} f_m (X')^m, \;\; G(X') \equiv \sum_{m=0}^{\infty} g_m (X')^m, \tag{2}$$

where infinite number sequences (f_m) and (g_m) ensure the norm convergence of their series. It is necessary to establish scalarization conditions for Eq. (1), i.e., conditions which allow one to reduce this equation to a set of scalar ordinary differential equations of the first order (ODE.1), including scalar equations (SE) ([1], P. 16)

$$x = tF(x') + G(x'). \tag{3}$$

2. Splitting of Lagrange matrix equations can be performed with the help of skew series introduced in [2]. They are used for defining several types of subalgebras in the full matrix algebra M_n; just over these subalgebras one considers Eqs. (1). We understand skew series of a matrix $A \equiv (a_{ij})$ as matrices $A_{\pm\alpha}$ ($\alpha = \overline{0, (n-1)}$) whose nonzero elements can be only $a_{i\,i\pm\alpha}$, i.e., matrices that contain only those elements of A that are located in a line parallel to its main diagonal. The subscript $\pm\alpha$ indicates the number of a line which is located either over $(+\alpha)$ or under $(-\alpha)$ the main diagonal. Evidently, any matrix in M_n is representable as follows:

$$A \equiv \sum_{\beta=1-n}^{n-1} A_\beta. \tag{4}$$

Skew series in the full matrix algebra allow one to define subalgebras of diagonal ($H_{n,0}$), upper triangular ($T_{n,0}$), upper nil-triangular ($T_{n,q}$), and striped ($M_{n/k}$) matrices. In [2] one also considers dual-skew series $A^*_{\pm\alpha}$ of matrices A. For them nonzero elements are only $a_{i\,n+1\mp\alpha-i}$. These matrices consist only of those elements of the initial matrix A that are located in lines parallel to its secondary diagonal, at the

*E-mail: p_derevenskiy@mail.ru.