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On Some Functional Calculus of Closed Operators in a Banach Space

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Abstract—We develop a functional calculus of closed operators in a Banach space based on the class of functions in the form $1/g$, where g belongs to the class $R[a, b]$ introduced by M. G. Krein. We prove continuity, stability, uniqueness, spectral mapping, and inverse operator theorems and describe some other properties of the considered calculus.

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1. INTRODUCTION AND PRELIMINARIES

Function classes $R[a, b]$ and $S[a, b]$ were introduced by M. G. Krein in connection with the moment problem and interpolation ones [1]. They are closely connected with classes $Q[a, b]$, $\widetilde{Q}[a, b]$, and $T[a, b]$ defined below. This paper is devoted to the development of a functional calculus based on the mentioned classes for unbounded closed operators in a Banach space. The main goal of the paper is to develop a calculus based on the class $Q[a, b]$; the rest calculi are auxiliary. Note that there are connections between functions of the mentioned classes which imply analogous connections between the corresponding functions of an operator argument; this allows us to treat the calculi under consideration in a unified way. Note also that the $Q[a, b]$-calculus appears to be useful for solving some classes of equations with closed operators [2].

We need some properties of functions from classes of Nevanlinna R and M. G. Krein $R[a, b]$ [1].

A function f is said to belong to the class R, if it is holomorphic in the open upper half-plane and maps it to itself.

Definition 1. Let $a < b$. A function g is said to belong to the class $R[a, b]$, if it belongs to R, is holomorphic and positive on $(-\infty, a)$, and is holomorphic and negative on $(b, +\infty)$.

Such a function g is uniquely representable as follows:

$$g(z) = \int_a^b \frac{d\tau(t)}{t - z}, \tag{1}$$

where τ is a bounded nondecreasing function different from a constant one (a "representing measure") ([1], pp. 525–526). Hence it easily follows that functions from $R[a, b]$ are holomorphic on $\mathbb{C} \setminus [a, b]$ and at infinity; they never vanish on $\mathbb{C} \setminus [a, b]$, and have at infinity a zero of multiplicity 1 [3]. Functions in the form (1) are also called the Markov functions.

In what follows, the symbol X stands for a Banach space over the field $\mathbb{C}$, and A denotes a closed unbounded operator in X, whose spectrum does not intersect with $[a, b]$. The symbol $\mathcal{F}(A)$ denotes the algebra of functions, each one of which is holomorphic in some neighborhood of the spectrum $\sigma(A)$ of the operator A and at infinity.

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