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Boundary-Value Problems for a Hyperbolic Equation with Nonlocal Conditions of the I and II Kind

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Abstract—In this paper we consider two initial-boundary value problems with nonlocal conditions. The main goal is to propose a method for proving the solvability of nonlocal problems with integral conditions of the first kind. The proposed method is based on the equivalence of a nonlocal problem with an integral condition of the first kind and a nonlocal problem with an integral condition of the second kind in a special form. We prove the unique existence of generalized solutions to both problems.

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1. The problems. In the domain $Q = (0, l) \times (0, T)$, where $l, T < \infty$, we consider the equation

$$u_{tt} - u_{xx} + c(x,t)u = f(x,t). \tag{1}$$

Problem 1. In the domain Q it is required to find a solution to Eq. (1) satisfying the initial data

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x) \tag{2}$$

and nonlocal conditions

$$\int_0^l K_i(x)u(x,t)dx = 0, \quad i = 1, 2. \tag{3}$$

Problem 2. In the domain Q it is required to find a solution to Eq. (1) satisfying the initial data (2) and nonlocal conditions

$$\begin{aligned} u_x(0,t) - \int_0^l K_1(x,t)u(x,t)dx = 0, \\ u_x(l,t) - \int_0^l K_2(x,t)u(x,t)dx = 0. \end{aligned} \tag{4}$$

Under conditions (3) and (4), $K_i(x)$ and $K_i(x,t)$ are defined in $\overline{Q}$ and sufficiently smooth (this is necessary for further transformations).

Problems with nonlocal integral conditions are actively studied now. It appears that the choice of the method for proving their solvability essentially depends on the kind of nonlocal conditions.

One usually understands *nonlocal conditions of the* II *kind* as correlations connecting values of the desired solution (in some domain Ω) and, possibly, its derivatives on some inner manifold and at boundary points of the domain Ω.

But if these correlations do not contain values of the desired solution on the domain boundary, then they are called nonlocal conditions of the I *kind*.

The intuition hints, and the practice of studying nonlocal problems confirms that problems with nonlocal conditions of the I kind lead to certain difficulties analogous to those in the theory of integral

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