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# DIFFERENTIAL TURNING POINT IN THE THEORY OF SINGULAR PERTURBATIONS. I

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## Introduction

The theory of singularly perturbed differential equations (SPDE) with turning points started its fast development from investigations by R. Langer (see [1]–[4]). Presently, basic results in the construction of uniform asymptotics for the Liouville equation and systems of SPDE of Liouville type with turning point are already obtained (see [1]–[14]). A characteristic peculiarity of the most of these works is that degenerate equations were algebraic, i. e., the turning point was algebraic.

However, the theory and the practice require the study of SPDE for which degenerate equations are differential and the turning point can be found in the form of a factor at the higher derivative of the degenerate equation.

The following equations represent classical examples of such equations

$$\mathbf{L}_\varepsilon y(x,\varepsilon) \equiv \varepsilon^3 y'''(x,\varepsilon) + x\widetilde{a}(x)y'(x,\varepsilon) + b(x)y(x,\varepsilon) = h(x), \tag{0.1}$$

$$\varepsilon y''(x,\varepsilon) + x\widetilde{a}(x)y'(x,\varepsilon) + b(x)y(x,\varepsilon) = h(x) \tag{0.2}$$

as well as the Orr–Sommerfeld equation

$$\varepsilon y^{(4)}(x,\varepsilon) + x\widetilde{a}(x)y''(x,\varepsilon) + b(x)y'(x,\varepsilon) + c(x)y(x,\varepsilon) = h(x). \tag{0.3}$$

A common feature of all these three classical equations is that all roots of the corresponding equations coincide at the point $x = 0$, i. e., at the turning point. At the same time, each of these equations possesses certain essential singularities which are characteristic only for such an equation. The investigation and construction of uniform asymptotics of these equations is a complex problem. The problem of construction of uniform asymptotics of such equations is known during several decades (see [4]; [13], pp. 225–230). As long ago as in 60s, S.A. Lomov for his disciples set the problem about investigation of equation (0.2) (Gordeyev's problem). However, fundamental results in this direction are not yet obtained up to now. Here it seems to be convenient to mention that by the concordance method in [14] (pp. 52–63) the asymptotic of solution of equation (0.2) on the segment $[0;1]$ was constructed.

For equations (0.1) and (0.3) in [2]–[4] certain results were also obtained. However, they were of a particular form, and further generalizations of these results were not obtained.

The aim of this article is a generalization of the method developed in [7]–[12] for investigation of scalar and vector equations with algebraic turning point onto equations with a differential turning point. The first step in this direction will be the investigation of a singularly perturbed differential equation (0.1).

Equation (0.1) is investigated for the case where the following conditions are fulfilled:

*Condition* $1^0$. $a(x), b(x), h(x) \in \mathbf{C}^\infty[I]$, $I = [0;1]$;