Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 45, No. 12, pp. 18–28, 2001 *UDC 519.854*

APPLICATION OF THEOREMS ON ALTERNATIVES TO DETERMINATION OF NORMAL SOLUTIONS OF LINEAR SYSTEMS

A.I. Golikov and Yu.G. Yevtushenko

1. Introduction

An ample bibliography is devoted to numerical methods for solving systems of equations and inequalities. In applications, the case where systems under consideration have no solution, is rather often encountered. In [1]–[4] problems of that kind, called improper, were investigated in detail and special means were suggested to correct the incompatibility. A simultaneous study of both compatible and incompatible systems can be met in various monographs, papers devoted to the theorems on alternatives (see, e. g., [5]–[10]). In these works, to every system of linear equations and inequalities they put into correspondence an alternative system such that each of both systems cannot possess a solution at the same time and only one of them is always solvable. It is not known a priori whether a given system possesses a solution. Therefore it is necessary, first, to clarify whether a given system is solvable and, second, to find its solution if it is solvable.

In this article we suggest an approach to solving systems of linear equations and inequalities on the basis of a constructive proof of theorems on alternatives. The substantiation of the approach suggested is carried out on the basis of the theory of duality of the nonlinear programming. We introduce systems conjugate and alternative to the initial one. The dimension of variables in these systems is equal to the number of equations and inequalities in the initial problem. The solving is reduced to an unconditional minimization of the norm of discrepancy of either initial, or alternative system.

If the initial system is given as well as the alternative one, then for the determination of a solution of a solvable system it suffices to minimize the discrepancy of one of these two systems. This problem supplies us with information more rich than a simple solution of the initial system. If for a chosen system after the unconditional minimization we obtain the zero discrepancy, then the solution of this problem is simultaneously a solution of the chosen system. In addition, one can affirm that the second system has no solution. If the minimal discrepancy of the chosen system differs from zero, then this system has no solution while the second system is solvable, and by the result of the minimization carried out via simple formulas one can calculate the solution of this system with the minimal norm (normal solution).

As in the approach used in [11], we formulate the problem which along with the problem of unconditional minimization of the norm of discrepancies form a pair of mutually dual problems. The apparatus of the duality theory allows us to carry out a simple constructive proof of the theorem on alternatives.

In the capacity of an example of the approach suggested, we consider non-traditional two-parametric condition of optimality for problems of linear programming (see [12]). We formulate an incompatible system possessing a lesser dimension, which is alternative to the system of the

Supported by the Russian Foundation for Basic Research, code of project no. 01-01-00804, and by the Program of State Support of Leading Scientific Schools, code of project no. 00-15-96080.