Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 8, pp.53–56, 1998 *UDC 519.716*

THE COMPLETENESS PROBLEM IN THE CLASS OF POLYNOMIALS IS RESOLVABLE

V.D. Solov'yov

The problem of completeness of finite sets of partially recursive functions and predicates is well-known (see [1]). The first complete sets were given in [2]. For instance, the set $\{0, x+1, =\}$ is complete. It was shown in [3] that the completeness problem is algorithmically unresolvable.

In the cases where for a class of objects a certain problem turns to be algorithmically unresolvable, one usually tends to select substantive subclasses of such a class of objects, for which the given problem will turn to be already resolvable.

In the present article we shall show that in the class of polynomials with integer coefficients the completeness problem is resolvable. Since the partially recursive functions traditionally are defined on the set of natural numbers N, we restrict the values of the polynomials to the set N. For an arbitrary polynomial f with integer coefficients, a function $\overline{f}(x_1,\dots,x_n) = \max(f(x_1,\dots,x_n),0)$ is called a polynomial on N.

Definition 1. The scheme of programs with arrays and equality is a finite oriented graph whose all nodes are marked with instructions.

The instructions can have the following form: a) begin $(x_1,\dots,x_n)$, b) $y := f(x_1,\dots,x_n)$, c) $y := x$, d) $x = y$, e) $p(x_1,\dots,x_n)$, f) $i := i+1$, g) $i := 0$, h) $i = j$, i) $M[i] := x$, j) $x := M[i]$, k) stop (x).

The syntax and semantics of the schemes of programs are standard (see [4]).

We shall denote by $FDA_=$ the class of schemes of programs from Definition 1.

For the set $\mathcal{L} = \langle f_1,\dots,f_n,p_1,\dots,p_m\rangle$, where $f_1,\dots,f_n$ are functions, $p_1,\dots,p_m$ are predicates, we denote by $[\mathcal{L}]_{FDA_=}$ the closure of $\mathcal{L}$ with respect to the class $FDA_=$, i. e., the set of all partially recursive functions computable by the schemes of programs from $FDA_=$ for substitution of functions and predicates from $\mathcal{L}$ instead of the variables f, p in the schemes of programs.

Definition 2. A set $\mathcal{L}$ is said to be complete (with respect to $FDA_=$) if $[\mathcal{L}]_{FDA_=}$ is the set of all partially recursive functions.

In [5] the following criterion for completeness was given: $\mathcal{L}$ is complete if and only if

1) $\forall x, y\ \exists\tau$ (τ is the term of the signature $\mathcal{L}$ & $\tau(x) = y$),
2) $\exists a\ \exists\Phi$ (Φ is quantor-free formula of signature $\mathcal{L}\cup\{=\}$ & $\Phi(a)$ & $\forall y \neq a \neg\Phi(y)$).

We shall use it in order to obtain the following result.

Theorem. *An algorithm exists which determines by any finite set of polynomials on N whether it is complete in the functional system $\langle\mathcal{F}_0, FDA_=\rangle$.*

Supported by Russian Foundation for Basic Research, the grants 93-011-16004, 98-01-00900.