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Application of the Principle of Symmetry to the Solution of the Slichter Problem

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We study the well-known problem on the filtration of a liquid to a system of drainage channels, whose bottom is situated at an impervious base. Assume that on the ground surface there is an infinitesimally thin sheet of water, while the water depth in channels equals h. Let T and $2L$ stand for the occurrence depth of the aquifuge and the distance between the channels, respectively.

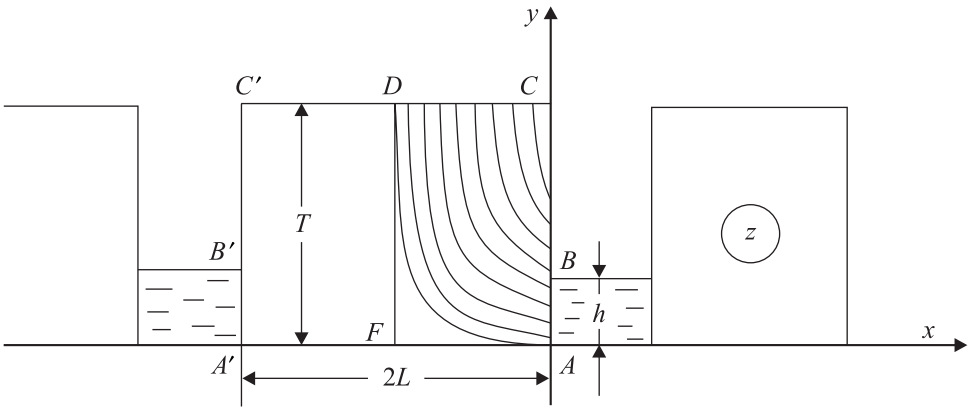

Fig. 1. Scheme of the flow

This problem was first studied in [1] under the assumption that the water depth in channels is negligible. However, the use of the Fourier solution method results in the following fact. The convergence of the Fourier series obtained for components of the filtration rate becomes very slow as the ratio L/T increases, i.e., in the most cases which are of interest for practice. In [2] (P. 181) one solves a similar problem with $h = 0$ by the method of conformal mappings. In [3] one solves the problem by the same method, taking into account the presence of the water in channels. However, the introduction of an auxiliary parametric variable for the conformal mapping of the flow domain and the plane of the velocity hodograph curve results in rather intricate formulas. For this reason, no numeric calculations were performed and the influence of the parameters onto the character of the flow was not studied.

Let us apply the Riemann–Schwarz symmetry method ([4], P. 147) and the known properties of elliptic functions ([5], pp. 69–73). Below we show that this allows us to avoid difficulties connected with the use of the method of conformal mappings and to obtain an exact analytic solution to the problem by comparatively simple calculations (using the known special or elementary functions).

1. Problem definition and its solution. Assume that the flow of the subterranean water obeys the Darcy law with the known filtration coefficient $\chi = \text{const}$, the water migrates through a homogeneous isotropic ground. The latter and the seepage water are assumed to be incompressible.

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