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# Realizability of Words in Mosaic Labyrinths

**V. I. Grunskaya**[1*]

[1] *Ul'yanovsk State University, Dimitrovgrad Branch, pr. Dimitrova 4, Dimitrovgrad, 433512 Russia*

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**Abstract**—In this paper we study mosaic labyrinths with the help of words generated by them in the alphabet of labels attached to arcs and vertices of a labyrinth. We consider the problem of the characterization of words generated by a labyrinth. We propose a constructive recognition criterion, it defines whether a word is generated by a labyrinth or not. We establish conditions under which a word can be generated by a unique labyrinth, by a finite number of labyrinths, or by infinitely many labyrinths.

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## INTRODUCTION

The problem of interaction of automata with different types of environments is one of the most important problems in automata theory. The language that originates from the automaton-environment interaction is the intersection of the automaton language and of that generated by the environment [1]. Therefore the problem of characterization of languages generated by environments becomes topical. Graphs with marked vertices and(or) arcs are considered as a topological model of an environment [2]. The languages generated by such graphs are naturally defined [3]. Labyrinths represent special cases of graphs. Traditionally a labyrinth is described geometrically as a subset of a set of points in the plane with integer Cartesian coordinates [4].

In this paper we study plane mosaic labyrinths. We propose to explore labyrinths by means of words generated by them in alphabets of arc and vertex marks. This approach allows one to do without the coordinate reference. We solve the problem of characterization of words generated by labyrinths. We develop a constructive criterion for recognizing words that are (or are not) generated in the indicated alphabet by labyrinths. We propose an algorithm for constructing a labyrinth that generates a given word with the least number of vertices. We establish criteria for generating words in a unique (to within isomorphism) labyrinth, in a finite number of labyrinths, or in infinitely many labyrinths. All undefined notions can be found in [4, 5].

## 1. BASIC DEFINITIONS

Let us consider the two-dimensional Euclidean space with the Cartesian system of coordinates and the integer lattice in it $Z^2$. We denote by $v$ elements $(x, y)$ of the set $Z^2$. We treat the number $\rho(v, v') = |x - x'| + |y - y'|$ as the distance between elements $v = (x, y)$ and $v' = (x', y')$. Elements $v$ and $v'$ are called neighboring if the distance between them equals one.

Let $B = \{e, n, s, w\}$ be an alphabet of arc marks. For $b \in B$ we use the notation $b^{-1}$ for the inverse mark. We assume that $e^{-1} = w$, $n^{-1} = s$, $w^{-1} = e$, and $s^{-1} = n$. Let $\theta$ be the empty mark, $B^0 = B \cup \{\theta\}$, and $A = 2^B \setminus \emptyset$.

A signed graph $G = (V, E, a, b)$ is defined as a plane directed finite connected graph, where $V \subseteq Z^2$ is the set of vertices, $E$ is the set of arcs, $a : v \in V \to a \in A$ is a vertex labeling function, and $b : (v, v') \in$

[*] E-mail: veragrunska@yandex.ru.