ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 2, pp. 66–73. © Allerton Press, Inc., 2010.
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An Over-Determined Boundary Problem for the Helmholtz Equation in a Semiinfinite Domain with a Curvilinear Boundary

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Received October 17, 2007

Abstract—In this paper we consider an over-determined Cauchy problem for the Helmholtz equation in a semiinfinite domain with a piecewise smooth curvilinear boundary. Applying the Fourier transform method in the space of distributions of slow growth, we establish the necessary and sufficient solvability conditions which connect the boundary functions. We construct integral representations of a solution.

DOI: 10.3103/S1066369X10020088

Key words and phrases: *over-determined Cauchy problem, Helmholtz equation, Fourier transform, space of distributions of slow growth.*

The wave scattering problem on a rough surface has become of importance in recent years due to its various applications in mathematical physics, including optics, radiowave propagation theory, and radiolocation ([1, 2]). This problem was studied by many researchers with the help of numerical and analytical methods (e.g., [3, 4]). In most cases the scattering problems were considered on a locally perturbed half-plane. In papers [5–7] one solved the scattering problem on a semiinfinite domain with a piecewise smooth boundary by methods of the potential theory.

In this paper we study an over-determined Cauchy problem for the Helmholtz equation in a semiinfinite domain bounded by a piecewise smooth curve. The two-dimensional problem of scattering of electromagnetic waves on a perturbed boundary of a half-plane is reduced to the problem mentioned above. Using the Fourier transform method in the space of distributions of slow growth, we obtain the necessary solvability conditions for the over-determined problem; they represent some dependence between boundary functions.

1. PROBLEM DEFINITION

Consider the Cauchy problem for the Helmholtz equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0 \tag{1}$$

with a real coefficient k in the domain D^+ (shown in Fig. 1 over the boundary S).

Let $S = \{(f_1(t), f_2(t)),\ t \in R = (-\infty, +\infty)\}$ be a piecewise smooth curve defined parametrically. Cosines of angles between its normal and coordinate axes are calculated by the formulas [8]:

$$\cos(n, x) = -\frac{g(t)}{\sigma(t)}, \quad \cos(n, z) = \frac{1}{\sigma(t)}, \tag{2}$$

where

$$\sigma(t) = \sqrt{1 + g^2(t)}, \quad g(t) = \frac{df_2(t)}{df_1(t)} = \frac{f_2'(t)}{f_1'(t)}. \tag{3}$$

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