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Integrodifferential Equations in Viscoelasticity Theory

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Abstract—We prove the correct solvability of the initial problems for integrodifferential equations with unbounded operator coefficients in Hilbert spaces. Such equations occur in many problems of the theory of viscoelasticity with memory and the heat transfer theory.

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In this paper we study integrodifferential equations with unbounded operator coefficients in a Hilbert space. Many problems arising in applications imply the investigation of indicated equations. First of all, they are problems of viscoelasticity theory in media with memory which take into account the Kelvin–Voigt instantaneous friction. At the present time, there are many publications dedicated to the study of such problems. Here we restrict ourselves to mentioning only the closest papers [1–6], where one considers integrodifferential equations with unbounded operator coefficients in a Hilbert space (see references therein).

In this paper we present results about the correct solvability of initial problems for the mentioned integrodifferential equations in the Sobolev weight spaces of vector functions that are defined on the positive semiaxis and take on values in a Hilbert space. We also consider the structure of the spectrum of operator functions which are the symbols of the indicated equations.

Let H be a separable Hilbert space and let A be a self-adjoint positive operator acting in the space H; we assume that A has a compact inverse operator. We transform the domain $\mathrm{Dom}(A^\beta)$ of the operator A^β, $\beta > 0$, into a Hilbert space H_β, introducing on $\mathrm{Dom}(A^\beta)$ the norm $\|\cdot\|_\beta = \|A^\beta \cdot\|$ equivalent to the norm of the graph of the operator A^β.

In what follows we use the denotation

$$u^{(n)}(t) := \frac{d^n u(t)}{dt^n}, \quad n \in \mathbb{N}.$$

We denote by $W^n_{2,\gamma}(\mathbb{R}_+, A^n)$ the Sobolev space of vector functions which are defined on the semiaxis $\mathbb{R}_+ = (0, \infty)$ and take on values in H with the norm

$$\|u\|_{W^n_{2,\gamma}(\mathbb{R}_+,A^n)} \equiv \left(\int_0^\infty e^{-2\gamma t} (\|u^{(n)}(t)\|^2_H + \|A^n u(t)\|^2_H) dt \right)^{1/2}, \quad \gamma \geq 0.$$

With $n = 0$ we set $W^0_{2,\gamma}(\mathbb{R}_+, A^0) \equiv L_{2,\gamma}(\mathbb{R}_+, H)$, and with $\gamma = 0$ we write $W^n_{2,0} = W^n_2$. See the monograph [7], Chap. I, for more details about spaces $W^n_{2,\gamma}(\mathbb{R}_+, A^n)$.

On the semiaxis $\mathbb{R}_+ = (0, \infty)$ we consider the following problem for a first order integrodifferential equation:

$$\frac{dv(t)}{dt} + \alpha A^2 v(t) + \int_0^t \mathcal{K}(t-s) A^2 v(s) ds = q(t), \quad t \in \mathbb{R}_+, \tag{1}$$

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