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FINITE-ELEMENT APPROXIMATION IN WEIGHT SOBOLEV SPACES

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1. *Introduction.* As is known, the problem of estimation of error of schemes of the finite element method applied for numerical solving boundary value problems of the mathematical physics is reduced to the problem of the theory of approximation concerning the estimation of the distance (in the norm of energetic space) between the solution of the boundary value problem and the space of finite elements. This estimate can be obtained via the construction of an appropriate for analysis operator of projection into the space of finite elements and by estimation of the norm of difference between the solution of the boundary value problem and its projection. The necessary condition for realization of such a scheme is the correct definition of the projector on the class of solutions with a reserve of smoothness, defined by the smoothness of the input data. Thus, the application of the classical interpolation operator into the space of finite elements requires the introduction of a function to be interpolated into the class C^r of r times continuously differentiable functions, where r is the maximal order of derivatives participating in the determination of the finite element (see [1], p. 62). This condition even with the use of the Lagrange finite elements ($r = 0$) can be restrictive due to an insufficient regularity of the approximated function, which takes place in problems with singular input data, for example, with degeneration of coefficients of the differential operator on boundary or on its part (see [2]). Therefore the construction of approximation procedures in conditions less restrictive than for a standard interpolation but not worse by approximating properties is of interest.

This article is devoted to the construction of a special operator of finite element approximation, which is defined in the Lebesgue space L_1, and obtaining estimates of error of this approximation for functions of weight Sobolev spaces. Estimates proved in this article are unimprovable, and in case of embedding of the approximate space into the space of continuous functions they coincide with the estimates established in [3] for the Lagrange n-simplexes with the use of the standard interpolation operator.

The procedure of approximation considered in this article is constructed with the use of mean values in common nodes of neighboring elements of local projections, i. e., projections onto finite elements, and allows us to combine various methods of local projection on different elements, satisfying the main weight estimate of the error on a finite element (Theorem 3). In particular, in using the orthogonal projecting operator in L_2 on an element, our approach is close to the Clement procedure (see [4]; also [1], p. 147), in which they use orthogonal projections on supports of functions of the canonical basis of the space of finite elements.

2. *Basic notions and notation.* For the sake of simplicity we restrict ourselves to exposition of results for the Lagrange n-simplexes. Let m be a natural number. By the Lagrange finite element of type (m) in R^n, or Lagrange n-simplex of type (m), we call the pair (K, ω_K), where K is a nondegenerate n-simplex in R^n with the vertices a_i, $i = \overline{1, n+1}$, while

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