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Reconstruction of Solutions to a Generalized Moisil–Theodorescu System in a Spatial Domain from their Values on a Part of the Boundary

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Abstract—In this paper we consider the problem of reconstructing solutions to a generalized Moisil–Theodorescu system in a spatial domain from their values on a part of the domain boundary, i.e., the Cauchy problem. We construct an approximate solution to this problem with the help of the Carleman matrix method.

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INTRODUCTION

We consider the problem on reconstructing a solution to the following system of equations [1]:

$$\operatorname{div} P + (P \cdot H) = 0, \quad \operatorname{grad} p_1 + \operatorname{rot} P + [P \times H] + p_1 H = 0, \tag{0.1}$$

which is a generalization of the Moisil–Theodorescu system ([2, 3], P. 169), from its known values on a part of the domain boundary, i.e., the Cauchy problem. As is known, the Moisil–Theodorescu system is a three-dimensional analog of the Cauchy–Riemann equations, whose significance in physical applications leads to many generalizations [4–6].

Introduce the following denotations: R^3 is the real three-dimensional Euclidean space;

$$x = (x_1, x_2, x_3), \quad y = (y_1, y_2, y_3) \in R^3, \quad x' = (x_1, x_2), \quad y' = (y_1, y_2) \in R^2,$$
$$\alpha^2 = (y_1 - x_1)^2 + (y_2 - x_2)^2, \quad r^2 = \alpha^2 + (y_3 - x_3)^2 = |y - x|^2;$$

Ω is a bounded simply connected domain in R^3; $\partial\Omega$ is its piecewise-smooth boundary consisting of a compact connected part T of the plane $y_3 = 0$ and a smooth Lyapunov surface S lying in the half-space $y_3 > 0$, $\overline{\Omega} = \Omega \bigcup \partial\Omega$, $\partial\Omega = T \cup S$; $p_1(x)$ and $P(x) = (p_2(x), p_3(x), p_4(x))$ stand, respectively, for a scalar and vector functions which have continuous derivatives of the first order in the indicated domain. We assume that any ray emanating from any point $x \in \Omega$ meets the surface S at most at l points. The symbol $A(\Omega)$ stands for the set of all vector functions of the class $C^1(\Omega)$ that are generalized-holomorphic in Ω; $H = (a_1, a_2, a_3)$ is a given constant vector.

A four-component vector $q = (p_1, p_2, p_3, p_4)$ is said to be *generalized-holomorphic* in Ω if it solves the elliptic system (0.1).

The Cauchy problem for the generalized Moisil–Theodorescu system, like many other Cauchy problems on the evaluation of regular solutions to elliptic equations, in general, is instable with respect to uniformly small perturbations of the initial data. Thus, these problems are ill-posed ([7], P. 39).

In fundamental monographs [8, 9] L. A. Aizenberg and N. N. Tarkhanov consider a regularization of the Cauchy problem for the Cauchy–Riemann system and for systems with injective symbols. One can find there an extensive bibliography.

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