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SOME REMARKS ON BOHR RADIUS FOR POWER SERIES

L.A. Aĭzenberg, I.B. Grossman and Yu.F. Korobeĭnik

1°. The classical Bohr's result (see [1]), reduced to its final form by M. Riesz, I. Schur and F. Wiener, is as follows: If the series

$$\sum_{k=0}^{\infty} c_k z^k \tag{1}$$

converges in the unit disk and its sum in this disk by its absolute value is less than 1, then in the disk $\{z : |z| < \frac{1}{3}\}$ the inequality $\sum\limits_{k=0}^{\infty} |c_k z^k| < 1$ is valid; moreover, the constant $\frac{1}{3}$ cannot be improved. In [2] another variant of Bohr's phenomenon was given: If the sum $f(z)$ of series (1) has in the unit disk a positive real part and $f(0) > 0$, then

$$\sum_{k=0}^{\infty} |c_k z^k| < 2f(0) \tag{2}$$

in the disk $\{z : |z| < \frac{1}{3}\}$. In addition, the constant $\frac{1}{3}$ in inequality (2) also is unimprovable. If $f(z)$ is the sum of series (1), then we define $Mf(z) := \sum\limits_{k=0}^{\infty} |c_k| z^k$ and the majorant function $Mf(r)$. In [3] the problem of determination of the quantity $\sup\limits_{f\in B_1} \inf\limits_{0<r<1} \frac{Mf(r)}{r} := A$ was posed, where B_1 is the class of all functions $f(z)$ analytic in the unit disk, for which $|f(z)| < 1$ with $|z| < 1$. From the Bohr theorem it follows that $A \le 3$. In [3] it was erroneously stated that $A = 3$. The correct result published in [4] says $A = 2$.

In addition, let us note some generalizations of Bohr's result for power series in the unit disk. Denote by $R_{(n)}$ the Bohr radius for the class of functions of the form

$$\sum_{k=n}^{\infty} c_k z^k, \tag{3}$$

i. e., the greatest radius such that, if the sum of series (3) belongs to B_1, then $\sum\limits_{k=n}^{\infty} |c_k z^k| < 1$ in the disk $\{z : |z| < R_{(n)}\}$. Obviously, $R_{(0)} = \frac{1}{3}$. In [5] it was proved that $R_{(n)} \ge b_n$, where b_n is the positive root of the polynomial

$$x^{n+1} + x - 1 + \frac{1}{4}x^{n-1}(1-x)^2.$$

In [6], for $q \in [1, 2)$, the following function

$$\alpha_q(r) = \sup_{f\in B_1} \Big[\sum_{k=0}^{\infty} \Big(\frac{|f^{(k)}(0)|}{k!} \Big)^q r^{qk} \Big]^{\frac{1}{q}}$$