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SOLUTION OF THE BASIC BOUNDARY VALUE PROBLEM FOR B-BIHARMONIC EQUATION BY THE METHOD OF POTENTIALS

M.Yu. Denisova

Let E_3^+ be a halfspace $x_3 > 0$ of the Euclidean space E_3 of points $x = (x_1, x_2, x_3)$, D a symmetric with respect to the coordinate plane $x_3 = 0$ domain bounded by the surface Γ. We denote by D^+ and Γ^+ the parts of D and Γ, receptively, situated in E_3^+. The domain D^+ is bounded by the surface Γ^+ and a part $\Gamma^{(0)}$ of the coordinate plane $x_3 = 0$. The surface Γ^+ is a surface of the class $\Lambda_{m,B}$, when $\Gamma \in \Lambda_m$ (see [1]).

In the domain D^+ we consider the equation

$$\Delta_B^2 u = 0, \tag{1}$$

where $\Delta_B = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + B_{x_3}$, $B_{x_3} = \frac{\partial^2}{\partial x_3^2} + \frac{k}{x_3}\frac{\partial}{\partial x_3}$ is the Bessel operator, k is an arbitrary positive number.

In this article we construct the fundamental solutions and potentials for equation (1), calculate the limit values of potentials on the boundary Γ^+, the basic boundary value problem for equation (1) is reduced to a system of the Fredholm integral equations of the second kind.

1. Potentials of simple layer and double layer types

As is known (see [2]), the fundamental solutions of equation (1) with a singularity at the origin are the functions $q_1(x) = r^{-k-1}$, $q_2(x) = r^{-k+1}$, where $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

To obtain the fundamental solutions with a singularity in an arbitrary point ξ, we apply to the functions q_1 and q_2 the generalized translation operator:

$$Q_1(x;\xi) = T_x^\xi q_1(x) = C_k \int_0^\pi \left((x_1-\xi_1)^2 + (x_2-\xi_2)^2 + x_3^2 + \xi_3^2 - 2x_3\xi_3\cos\varphi\right)^{\frac{-k-1}{2}} \sin^{k-1}\varphi\, d\varphi,$$
$$Q_2(x;\xi) = T_x^\xi q_2(x) = C_k \int_0^\pi \left((x_1-\xi_1)^2 + (x_2-\xi_2)^2 + x_3^2 + \xi_3^2 - 2x_3\xi_3\cos\varphi\right)^{\frac{-k+1}{2}} \sin^{k-1}\varphi\, d\varphi,$$

where $C_k = \frac{\Gamma(\frac{k+1}{2})}{\sqrt{\pi}\Gamma(\frac{k}{2})}$. Using the scheme suggested in [3], one can easily show that the fundamental solutions Q_1 and Q_2 can be represented in the form

$$Q_1(x;\xi) = \frac{C_k (x_3\xi_3)^{\frac{-k-1}{2}}}{2}\frac{1}{r_{x\xi}} + \psi_1(x;\xi), \quad Q_2(x_1;\xi) = \frac{C_k (x_3\xi_3)^{\frac{-k-1}{2}}}{2} r_{x\xi} + \psi_2(x;\xi),$$

where ψ_1 and ψ_2 are regular parts of the solutions Q_1 and Q_2, respectively, and $r_{x\xi}$ is the distance between the points x and ξ.