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A Stability Criterion for a Difference Scheme in a Nonlocal Heat Conduction Problem

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We consider a difference scheme with weights which approximates a heat conduction equation with a nonlocal boundary condition. We obtain the necessary and sufficient stability conditions with respect to the initial data in a certain specially constructed energetic norm. The stability of difference schemes for nonlocal heat conduction problems was investigated earlier in papers [1–5]. The newness of this paper consists of the fact that the stability conditions cannot be weakened owing to the choice of the norm.

1. An abstract scheme with weights. The theory of stability of two-level and three-level difference schemes, which represent operator-difference equations in Euclidean spaces, is developed in papers of A. A. Samarskii (see the references in [6]). Any two-level difference scheme admits the canonic form

$$B\frac{y_{n+1}-y_n}{\tau}+Ay_n=0,\quad n=0,1,\dots,\quad y_0 \text{ is given,} \tag{1}$$

where $y_n = y(t_n) \in H$ is a function of a discrete argument $t_n = n\tau$, whose values belong to a finite-dimensional linear space H; A, B are linear operators, acting in H.

In what follows, we assume that the operators A and B are independent of n (constant operators). Further we never specially mention this fact. The possibility to proceed to variable operators is substantiated, for example, in [7] (P. 122). We also assume that the operator B is invertible and, therefore, Eq. (1) is uniquely resolvable with respect to y_{n+1}. Let a scalar product (y, v) and the norm $\|y\| = \sqrt{(y,y)}$ be defined in H. Let a self-conjugate positive operator $D : H \to H$ be given. We call the norm $\|y\|_D = \sqrt{(Dy,y)}$ the energetic norm, we do the operator D the norm operator. We understand the space H_D as the set of all elements $y \in H$ with the norm $\|y\|_D$. A difference scheme is said to be stable in the space H_D, if with any initial data $y_0 \in H$ its solution meets the inequalities

$$(Dy_{n+1}, y_{n+1}) \leqslant (Dy_n, y_n),\quad n=0,1,\dots$$

This definition implies that the stability takes place in H_D if and only if the following operator inequality is true:

$$D \geqslant S^*DS, \tag{2}$$

where $S = E - \tau B^{-1}A$ is the transition operator of scheme (1), S^* is its conjugate operator, and E is the unit operator.

In this paper we consider the scheme with weights

$$\frac{y_{n+1}-y_n}{\tau}+\sigma Ay_{n+1}+(1-\sigma)Ay_n=0,\quad n=0,1,\dots,\quad y_0 \text{ is given,} \tag{3}$$

which represents a partial case of scheme (1), when $B = E + \sigma\tau A$. Here σ is a given real parameter and A is a linear operator in H. Assume that an independent of n operator $D = D^* > 0$ is given.

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