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EXTREMAL PROPERTIES OF SOLUTIONS OF THE DIFFERENCE TRICOMI PROBLEM FOR ONE GRID SYSTEM OF MIXED TYPE EQUATIONS AND THEIR APPLICATIONS

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1. Problem definition. Main results

Consider the system

$$L_i U \equiv K(y)u_{ixx} + u_{iyy} + a_i(x,y)u_{ix} + b_i(x,y)u_{iy} + \sum_{j=1}^{n} c_{ij}(x,y)u_j = f_i(x,y), \tag{1}$$

where $K(y) = |y|^\beta \operatorname{sgn} y$, $\beta > 0$, $i = \overline{1,n}$, $n \geq 2$, $U = (u_1, u_2, \dots, u_n)$, in certain domain D. The latter is assumed to be bounded for $y < 0$ by characteristics AC, BC of system (1) originating from points $A(0,0)$ and $B(l,0)$, $l > 0$; and for $y > 0$ by a simple curve σ with end points at A and B. We denote the parts of the domain D, where $y > 0$ and $y < 0$, correspondingly, by D^+ and D^-.

Consider for system (1) in the domain D the following boundary value problem.

The Tricomi problem. Find a function $U(x,y)$ which satisfies the conditions

$$U(x,y) \in C(\overline{D}) \cap C^1(D) \cap C^2(D^+ \cup D^-); \tag{2}$$

$$L_i U(x,y) \equiv f_i(x,y), \quad (x,y) \in D^+ \cup D^-, \quad i = \overline{1,n}; \tag{3}$$

$$U(x,y) = \Phi(x,y), \quad (x,y) \in \sigma; \tag{4}$$

$$U(x,y)\big|_{AC} = \Psi(x), \quad 0 \leq x \leq l/2, \tag{5}$$

where $\Phi = (\varphi_1, \varphi_2, \dots, \varphi_n)$ and $\Psi = (\psi_1, \psi_2, \dots, \psi_n)$ are given sufficiently smooth vector-functions, $\varphi_i(0,0) = \psi_i(0)$.

Traditionally the Tricomi problem for mixed type equations and systems is studied by the analytical methods. For example, in [1], [2] a new analytical solution method for the Tricomi problem for the Lavrent'ev–Bitsadze equation is proposed. In [3], the maximal principle for the module $|U(x,y)| = \sqrt{\sum_{i=1}^{n} u_i^2(x,y)}$ of a solution of Problem T for system (1) is established. It is done assuming that $K(y) = y$, $a_i(x,y) = b_i(x,y) \equiv 0$, $(c_{ik}(x,y))$, $i,k = \overline{1,n}$, $n \geq 2$, is a negatively defined matrix, whose components in the domain D^+ satisfy the conditions $*$:

$$(n-1)(c_{ik}(x,y) + c_{ki}(x,y)) \leq 2(c_{ii}(x,y)c_{kk}(x,y))^{1/2}, \quad i \neq k,$$

and in the domain D^- they are sufficiently small. This principle implies the uniqueness of the solution of Problem T. Using the uniqueness theorem, by the method of integral equations the existence theorem is proved for a regular solution of the Tricomi problem, assuming that the

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