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On the Structure of a Solution Set of Controlled Initial-Boundary Value Problems

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Abstract—For a controlled nonlinear functional-operator equation of the Hammerstein type describing a wide class of controlled initial-boundary value problems, we obtain simple sufficient conditions for the convexity, pointwise boundedness and precompactness of the set of solutions (the reachability tube) in the Lebesgue space. As for boundedness and precompactness, we mean certain conditions of the majorant but not Volterra type requirements which give also the total (with respect to the whole set of admissible controls) preservation of solvability of mentioned equation. As some examples of reduction of a controlled initial-boundary (boundary) value problem to the equation under investigation, and verification of the proposed hypotheses for this equation, we consider the first initial-boundary value problem associated with a semilinear parabolic equation of the second order in a rather general form, and also the Dirichlet problem associated with a semilinear elliptic equation of the second order.

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INTRODUCTION

The information about geometrical and topological structure of sets and tubes of reachability for controlled *initial-boundary value problems* is very important in the investigation of a variety of control problems. For detailed clarification on this account, see [1], which has historical and bibliographical references. The present paper is a continuation of the investigation [1] concerning mostly convexity and precompactness conditions for reachability tubes of controlled distributed systems. In this regard, we will note the following.

For semilinear systems with essentially nonlinear right-hand side, sufficient conditions for the convexity of reachability sets known to the author (except from ones found in [1]) have local nature, even in condensed case. In particular, it is required either for a control to belong to a ball in L_2 of sufficiently small radius, or final time to be small, or (in case of control in initial conditions) initial state to belong to a sufficiently small ball (e.g., [2, 3]). In [4], the following example of a non-convex reachability set is given:

$$x_1'(t) = -(x_2)^2 + u_1(t), \;\; x_2'(t) = u_2(t), \;\; t \in (0;T];$$

$$u_j(t) \in [-1;1], \;\; t \in [0;T], \;\; j = 1,2.$$

Here the set of reachability from zero to any positive time T is not convex. This example gives us an idea that non-local sufficient convexity conditions for reachability set of controlled systems (even condensed), nonlinear in phase variable, should have some special form. The conditions found in [1] were rather specific.

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