ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2014, Vol. 58, No. 3, pp. 60–63. © Allerton Press, Inc., 2014.
Original Russian Text © D.V. Kurlikovskii, 2014, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2014, No. 3, pp. 72–76.

Methods of Localization of Discontinuities in Solution of First Kind Equation of Convolution Type

D. V. Kurlikovskii[1*]
(Submitted by V.V. Vasin)

[1] *Ural Federal University, ul. Turgeneva 4, Ekaterinburg, 620075 Russia*
Received September 18, 2013

Abstract—We consider a convolution-type equation of the first kind from L_1 to L_1. We construct and investigate regularizing methods for localization (detection of positions) of discontinuities of the first kind of solution to this equation. Under additional conditions of exact solution, we obtain estimates for the accuracy of localization and for the separability threshold, which are another important characteristics of the methods.

DOI: 10.3103/S1066369X14030074

Keywords and phrases: *ill-posed problem, localization of singularities, regularizing algorithm, separation threshold.*

1. INTRODUCTION

Let us consider the convolution-type integral equation of the first kind

$$Ax \equiv \int_{-\infty}^{+\infty} K(t-s)x(s)ds = y(t), \quad t \in (-\infty, +\infty). \tag{1}$$

In what follows we denote by L_1 and $\|\cdot\|_{L_1}$ $L_1(-\infty,+\infty)$ and $\|\cdot\|_{L_1(-\infty,+\infty)}$. We assume that the functions K, x belong to L_1. Therefore, by theorem 1.3 from [1] (P. 9) (with $p = 1$) the function y also belongs to L_1 and the operator A is a continuous operator acting from L_1 to L_1. Let we know the following a priori information with respect to the exact function x:

(A) the function x has an unknown number l of the first kind discontinuities at unknown points $\{s_k\}_1^l$;

(B) outside discontinuity points the function x is continuously differentiable, and at each discontinuity point there exist left and right finite limits of the derivative, the function x and its derivative x' belong to L_1: $\|x\|_{L_1} \le r$ and $\|x'\|_{L_1} \le r$ (at points s_k the function x' is arbitrarily redefined);

(C) numbers $L > 0$, $\Delta^{\min} > 0$, and $\Delta^{\max} > 0$ are given such that $0 < l \le L$, $\min\{|\Delta_k| : k = 1, 2, \dots, l\} \ge \Delta^{\min}$ and $\max\{|\Delta_k| : k = 1, 2, \dots, l\} \le \Delta^{\max}$, where $\Delta_k = x(s_k + 0) - x(s_k - 0)$.

In condition **(B)** without loss of generality we can assume that the number r equals unity (as well in what follows).

Problem. By the function y^δ and an error level δ such that $\|y - y^\delta\|_{L_1} \le \delta$, it is required to define the number l and approximate states $\{s_k\}_1^l$ with the estimate of approximation accuracy.

*E-mail: dvkurlikovskiy@mail.ru.